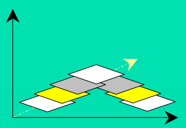
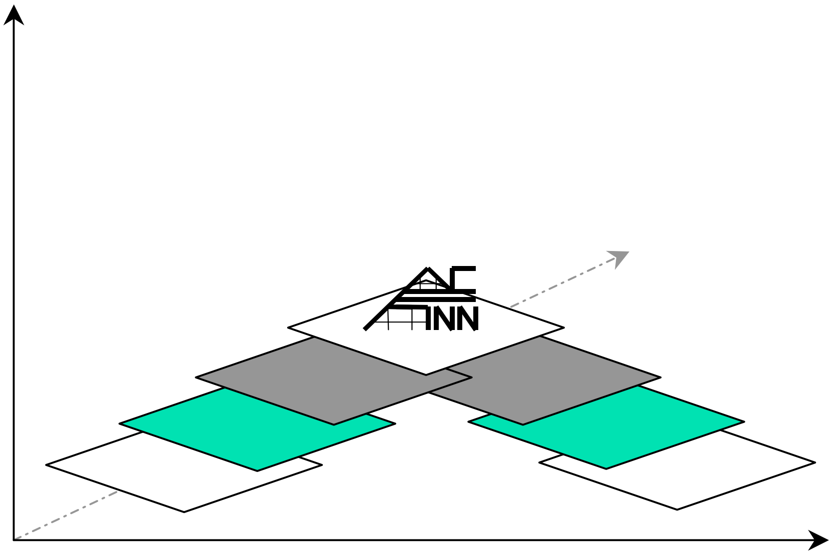


VALUE IN THE PROCESS OF REAL ESTATE MANAGEMENT AND LAND ADMINISTRATION



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VALUE IN THE PROCESS OF REAL ESTATE MANAGEMENT AND LAND ADMINISTRATION

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T O W A R Z Y S T W O N A U K O W E N I E R U C H O M O Ś C I

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INTRODUCTION

Managing real estate resources, especially the public ones, has a significant impact on national economy, as the resources owned by the State Treasury and local governments totals to over 40% of all real estate in Poland.

Proper and efficient system of real estate management should include, among others:

- facilitation of reshaping area structures,
- assistance in natural environment management and protection of valuable areas,
- development and monitoring real estate market,
- the best use of gathered real estates considering the rules of sustainable development (economy, environment, society).

Land, the basic natural resource, is considered both as material and abstract good that includes rights (ownership, allocation, and way of use), physical attributes (aptitude to various functions) and specific characteristics. Land ownership structure, its value and ways of real estate use are practically quite dependent of one another.

Every land's attribute should be unambiguously identified, registered and taken into consideration.

Real estate management should result in the most rational use of real estate resources. It demands mainly defining standards related to the management, studying and identifying basic problems occurring within this system and detailed analysis within particular procedures. These analyses are based mostly on the results of various specific studies with the use of mathematical methods of statistics.

In the frames of the economic view, real estate is usually treated as the object of distribution among various subjects and ownership sectors.

Both economy and real estate management require efficient information on the object of management including the possibility to identify different occurrences.

Real estate management is usually connected with:

- legal regulations and restrictions relating to the access to particular area,
- changing economic conditions,
- administrating and managing the rules of real estate resources development,
- managing the system of area information.

Important activities connected with the management include the analysis of real estate ownership structures, estimation of resources components and descriptions of use structures in dynamic aspect.

Real estate resources management should increase the efficiency of its use regarding economic and environmental rules.

Active real estate management should be efficient and proper, no matter who is in charge of managing - the owner or licensed real estate manager. Making the appropriate decisions requires taking into consideration the results of real estate market analysis and real estate valuations.

Real estate valuation is the activity aimed at helping to achieve the goal of suitable managing. Every decision, including these concerning real estate, is quite often connected with the necessity to choose among few alternative possibilities. Current or future real estate owner usually demand information and data on expected real estate development and economic effects of business activities.

The current owner usually needs some advice with solving the dilemma – whether to continue present activities on the real estate, improve its conditions or maybe sell it and invest money in some other project with higher expected profits. However, the potential buyer (investor) requires help to choose the particular market segment, in which invested money brings faster return and higher profit.

In making above mentioned decision the real estate valuer can be helpful mainly to:

- set tenancy fees on the profitable level,
- define the possibilities of building and renovating objects,
- verify the correctness of financial documentation,
- define closing down value in the case of forced sale or auctioning,
- give some investment advice,
- present probable results of alternative solutions,
- study market trend of supply and demand to provide the investor with reliable data relevant to make a decision where to allocate money, i.e. real estate, securities, etc.

Therefore the purposes of real estate valuation can be divided into two groups:

- static – when valuation does not serve changing real estate value, except the value can be substituted for price set on the real estate market,
- dynamic – when real estate value is defined in order to assess the highest limit to which it can be changed.

According to Polish legal regulations „if there is a need for assessing real estate value, this value has to be defined by the valuer" (Article 7, Act on Real Estate Management). Defining real estate value means assessing the value of ownership right or other rights to real estate.

The articles in this monograph mainly focus on:

1) defining types of value in Polish and international valuation standards, underlining market value and group of values other than market value;
2) quantitative and qualitative methods of assessing real estates similarities as a crucial issue in comparison approach;
3) creating the model of commune's influence on real estate market through given tools and instruments, both legal and formal;
4) analyzing the rules regulating paying the fees for perpetual usufruct for land owned by the commune (ways alternative to standard rules);
5) studying time series of real estate market with identification of non-stationary type of housing real estate price on Polish market;

6) defining, on the basis of transactions agreed on particular real estate market, the importance of particular real estate attributes in order to set the function of the area (applying approximate set theory).

Above mentioned issues bring closer the rules of real estate resources management including chosen detailed procedures. They are touching both valuation, communes' impact on real estate local market, and application of some statistical methods for the purpose of real estate management. Therefore, they are relevant to the studies on efficiency of real estate resources management and contribute to improvement of real estate resources management process both on the national and local level.

dr hab. Antoni Sobczak, prof. UE
prof. dr hab. inż. Ryszard Źróbek, prof. zw.
Scientific Editors

CHAPTER 1

REAL ESTATE VALUATION IN GLOBAL PERSPECTIVE

1.1. TYPES OF REAL PROPERTY VALUE IN POLISH, INTERNATIONAL AND EUROPEAN VALUATION STANDARDS

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Key words: *valuation standards, value of land and buildings, real estate value*

Abstract

Real estate value has been defined in international, European and national valuation standards and it refers to durable property, especially to real estate. All types of value have been divided in two basic groups – market value and other than market value. The latter group (non-market values) includes the following values: investment (or individual), fair, mortgage lending, forced sale, insurable, amortized reconstruction cost (depreciated replacement cost), and cadastre (fiscal). In international and European valuation standards special and synergic (marriage) values are also mentioned as particular types of fair value, and highest and best and alternative use value can be found there as the assumptions for market value estimation. Moreover, EU directives and national regulations have introduced additional own value definitions depending on the use of particular type of value in valuation process.

1. Introduction

Valuation means assessing the value for particular purposes and is strictly related to the type of the assignment, i.e. the value of the specific right to real estate in order to take the decision on sale or the value of ownership right to land without its integral parts in order to set the amount of planning fee.

As the value is assessed by the valuer or other licensed person, the purpose of valuation is formulated by the user of valuation services.

In valuation process the proper knowledge of valuation client's needs is essential to decide if it has to meet legal regulations and professional standards or it is just a part of property consulting (investment consulting) and analysis do not have to result in specific value. Thus, taking an assignment demands unambiguous definition of the value resulting from the particular valuation. Recently the number of orders on valuation and studies not being valuation report has been constantly increasing and, unfortunately, the same tendency can be observed with the number of matters of argument directed to opinion groups at valuers' associations or to Professional Responsibility Commission. In such situation the need arises to intensify activities leading to minimizing the risk of calling the authors to account and the risk of material damages to the receivers of these elaborates.

Thus there is a need for clear legal regulations normalizing the valuation issue and detailed rules of good professional practice.

In this article the various types of real estate value are presented, which definitions are regulated or just being regulated in Polish and international valuation standards. International Valuation Standards Council – IVSC in 2007 updated the International Valuation Standards – IVS and the European Group Of Valuers Association – TEGOVA prepared in 2008 the project of European Valuation Standards – EVS actualization. Moreover, on the national level, Polish Federation Of Valuers' Associations (PFVA) enacted major Common General Rules Of Valuation (Powszechne Krajowe Zasady Wyceny – PKZW) in 2007 and 2008. In the article, standards prepared by these organizations defining or interpreting term 'value' have been used.

2. Types of real estate value

According to IVS, 'value' is an economic term relating to the most probable price of article or service agreed on the market by buyers and sellers. Therefore the value can not be considered as a fact, but estimated price agreed in given moment and depending on chosen value definition. Economic concept of the value reflects the advantages resulting from the purchase of an article or service on the date of valuation (IVS 2007). According to these records, term 'value' means hypothetic price and its hypothesis depends on the valuation base (type of value).

On the basis of already mentioned valuation standards, all types of value can be divided in two main groups: market value and non-market value or other than market value.

In table 1, the values defined in IVS, EVS and PKZW are compared. Additionally, types of value taken into consideration in Polish legal regulations are added.

3. Market value as the main base of valuation

Many ideas how to define the meaning of 'market value' can be found in literature and written regulations, but generally it is assumed that market value is associated with the result of market observations and general analysis of market participants' behaviour. In Poland, this type of value is defined in the Act on Real Estate Management in the following way: *real estate market value means the most probable market price resulting from analysis of transactional prices, assuming that:*
1) parties of transactions were entirely independent of one another, acted without compulsion and were willing to draw up an agreement;
2) time necessary to expose the real estate on the market and to negotiate the agreement had passed (proper marketing).

International Committee for Valuation Standards formulated following definition of market value: *the estimated amount for which a property should exchange* *on the date of valuation between a willing buyer and a willing seller in an arm's-length transaction after proper marketing wherein the parties had each acted knowledgeably, prudently, and without compulsion* (IVS – Basic Definitions for Commonly Used Valuation Rules).

Table 1

Types of real estate value

<table>
<tr><th></th><th>According to IVS</th><th>According to EVS</th><th>According to PKZW</th><th>According to the legal regulations</th></tr>
<tr><td>Group I</td><td colspan="4">Market Value</td></tr>
<tr><td>Group II</td><td colspan="4">Other than market value</td></tr>
<tr><td>Category I</td><td>investment or individual</td><td>investment or worth</td><td>individual or investment</td><td>Adjusted to the needs of particular investor</td></tr>
<tr><td rowspan="4">Category II</td><td colspan="4">fair</td></tr>
<tr><td>special</td><td>special</td><td></td><td></td></tr>
<tr><td>synergistic</td><td>marriage (synergistic)</td><td></td><td></td></tr>
<tr><td>forced sale</td><td>forced sale</td><td>for forced sale</td><td></td></tr>
<tr><td rowspan="4">Category III</td><td>mortgage-lending</td><td>Mortgage - lending</td><td>Mortgage - lending</td><td>Mortgage - lending</td></tr>
<tr><td>tax (fiscal)</td><td></td><td>cadastre</td><td>cadastre</td></tr>
<tr><td colspan="4">insurable</td></tr>
<tr><td>depreciated replacement cost</td><td>depreciated replacement cost</td><td>depreciated replacement cost</td><td>depreciated replacement cost</td></tr>
<tr><td></td><td></td><td></td><td>cost</td><td>cost</td></tr>
</table>

Source: Own study on the basis of IVS, EVS, PKZW and relevant Acts.

These definitions presume that existing market brings together the activities and motivations of many its participants.

For the purpose of gradual adjusting the definition used in Poland to definitions written down in IVS and EVS, and also to standardize these fundamental principles of real estate value interpretation used in National Basic Valuation Standard 1 (Krajowy Standard Wyceny Podstawy 1 Wartość rynkowa i wartość odtworzeniowa – KSWP1) 'market value and replacement value', some modifications, such as 'parties of agreement are entirely independent of one another, act without compulsion and are willing to draw up a contract', need to be introduced. The interpretation of particular components of this definition has been adjusted to interpretation presented in IVS.

The same definition is accepted by tegova and placed in standard 1 market value.

In order to estimate market value, the valuer should at first decide on the highest and best use or the most probable use of the real estate. Such use can be the simple continuation of previous use or can be the alternative one. According to IVS, choosing the type of use should be made on the basis of market information and the most common methods used here are the comparison approach, income capitalization approach including discounted cash flows analysis and cost approach.

As it is stated in KSWP1, accepted type of use depends also on the aim of the valuation, because many official decisions in Poland are made in reference to market value that is estimated assuming i.e. real estate's actual use. However, according to the Act on Real Estate Management, market value can be estimated also with the use of comparison, income or, conditionally, mixed approach.

Again, in order to gradual standardisation of valuation methods, KSWP1 records state that: 'in special cases, after taking into consideration the type of real estate, purpose of valuation and actual condition of real estate market, replacement value resulting from the use of the cost approach can be the substitution for market value only if realization of methods and procedures designed for such approach is based on market data, both in reference to market value of land, costs of building and actual wear and tear'.

It should be added here, that in the EU directives many references to 'market value' can also be found. Most of them apply to the valuation of financial instruments and business, but there are some definitions including real estate. Generally, they reflect the same concept of the market value that is accepted by organizations preparing valuation standards. Since definitions were created for some specific purposes, the detailed records include additional, important for given case (given purpose), rules related to valuation, i.e. the necessity for proper documentation and clear justification for chosen estimation approach demanded by lending institutions. Directive 91/674/EEC on the annual accounts and consolidated accounts of insurance undertakings can be the next example. According to these regulations, if land and buildings were sold in the day of preparation of financial account, or would be sold in the nearest future, market value of real estate should be reduced by carried (or anticipated) transaction costs. In this case, the valuer has to precise in the report if assessed value includes these costs. In another way, the valuer has to define two real estate values: before and after taking into consideration above mentioned costs.

Another definition accepted by European Union for financial reports prepared by institutions operating within EU describes it as a price, which is probable to be paid by the purchaser after taking into consideration the condition and localization of given real estate and assuming that this asset can be still used in the same way.

3. Valuation basis other than market value

3.1. Investment (individual) value

Concepts of investment and market value differ in many respects. Investment value means price paid by particular investor for particular real estate treated as investment. Estimation of this value needs an application of investment criteria, which can be pointed by the investor or defined by the valuer after recognising the demands and investment purposes of his client.

This type of value is called 'value for the needs of individual investor'. According to Act on Real Estate Management, this value results from analysis and calculations and should be presented in study other than valuation report. Such records cannot be found both in international and European valuation standards.

3.2. Fair, specific, synergistic and forced sale value

The term 'fair value' appears in legal regulations and valuation standards in many countries, usually in connection with the financial reporting purposes. It is defined i.e. in International Financial Reporting Standards created by International Accountancy Standards Council as *estimated amount, which is equivalent to particular component of assets exchanged among well-informed, independent of one another and willing parties of transaction* and applied also in IVS.

According to NBVS2- Krajowy Standard Wyceny Podstawowy 2 (KSWP2) , fair value means the amount of money paid for component of assets exchanged in market transaction among interested, well-informed and independent parties. This phrase is consistent with above cited definition, National Specific Standard 2 Valuation for Financial Accounts Purposes and the Act on Accountancy – Krajowy Standard Wyceny Specjalistyczny 2 (KSWS2).

While studying above mentioned definitions it can be noticed that the fair value is usually equivalent to market value estimated with the use of the comparison approach. When the number of market transactions concerning comparable units is insufficient, this value can be assessed with the income approach. Conditionally, when data on market rent is also unavailable, the fair value results from the use of the cost approach. However, in international and European valuation standards the use of the cost approach leads to estimation of market value, but it can not be done according to Polish regulations.

For purposes other than financial reporting, the fair value can differ from the market value, for example, when the purpose of valuation is to assess the price which would be fair for two particular parties. Such situation needs taking into consideration in valuation procedure both positive and negative results of buy/sale transaction for each party. Nevertheless, certain component of assets can have different value for particular real parties than for hypothetic seller and buyer. Therein specific or synergic value can be applied (IVS2 for purposes other than financial accounts), which in EVS are respectively called special or marriage value.

Because of international application of this value in conditions of progressive business units' activities globalization it was decided to pay more attention to it in this article.

As it is presented in table 1, this category includes also value for forced sale. According to KSWP2, value for forced sale means the amount of money, which is paid for the real estate sold in compulsory circumstances. In the interpretation of this definition is stated that while estimating the value for forced sale, the valuer has to take into consideration that seller was under compulsion and it resulted in insufficient market exposition. When a real estate is not properly marketed, the real market price is impossible to be achieved.

It is stated in EVS that the value of forced sale can be the base for valuation only when all the circumstances of transaction are well known. Then it can be treated as the market value assessed with special assumptions.

However, in IVS the term 'forced sale' is regarded as the description of sale circumstances rather than the kind of value.

Next value with international references is mortgage-lending value.

3.3. Mortgage-lending, cadastre, insurable and reconstruction value

The mortgage-lending value is not mentioned in IVS2. Thus, it should be regarded as belonging to the third group of non-market values - values defined by legal regulations or agreements (contracts). However, Committee has published so called International Valuation Applications 2 – Valuation for Secured Lending Purposes. In these Applications the definition from EU Directive 2006/48/WE of 14 June 2006 is cited*: the value of the property as determined by a prudent assessment of the future marketability of the property taking into account long term sustainable aspects of the property, the normal and local market conditions, and the current use and alternative appropriate uses of the property. Speculative elements shall not be taken into account in the assessment of the mortgage lending value. The mortgage lending value shall be documented in a clear and transparent manner.*

TEGoVA regards this value as the base for valuation other than market value and comments it widely in European Valuation Standards Applications 2 (EVA 2). Polish legal definition of this value, which is written in Act on Legal Lien and Mortgage Banks, is strictly related to EVA 2. However, KSWP2 states that it should be considered as the amount of money equitable to the level of risk associated with the real estate as the subject of security for lending. It is underlined that this kind of value reflects the more stable value in long term comparing to the market value, which level is assessed on the date of valuation.

In standard KSWP2, the records concerning cadastre value, which is regulated in Act on Real Estate Management, can also be found. In updated IVS this kind of value has its equivalent called 'tax value' or 'fiscal value' and their interpretation are placed in IVA. The definition of cadastre value in KSWP2 merges the basic assumptions emerging from Polish law and IVS: *real estate cadastre value consists of*

land and its components cadastre value and is as similar to market value as it is possible to be achieved with the use of procedures typical for mass valuation methods.
These records have not been introduced to EVS.
Insurable value is not directly mentioned in IVS2. Thus, according to table 1, it can be ranked as the part of third category – the values defined by legal regulations or agreements signed with valuation service's client. However, it is mentioned in EVS2 as the amount of insurer responsibility fixed in real estate insurance policy.

KSWP2 defines this type of value as: *the amount of money assessed by the valuer for insurance object that is equivalent to upper limit of insurer responsibility for damages on real estate resulting from causes specified in the insurance policy*.

It can also be found in Polish valuation standard that insurable value, depending on object and scope of insurance, can be identified with various types of value which are defined in relevant legal regulations or in general conditions of insurance used in particular insurance company.

The last type of value characterised in this article is the reconstruction value, which is commonly used in Polish valuation practice and still functioning in legal regulations. This value is defined by KSWP1 as the amount of money equivalent to reconstruction costs of depreciated real estate, assuming that these costs were paid in the day of valuation. As some kind of equivalent described in EVS can be also assumed the term 'depreciated replacement cost', known also as 'contractor's method', but it is more commonly used in estimating market value with insufficient amount of market data.

In International Interpretation Guidance number 8 IVS the term of amortised reconstruction cost can be found.

The last two terms listed in EVS refer to highest and best use value and alternative use value and they were defined through the way of real estate use.

Highest and best use – according to IVS it is the basic assumption for assessing market value, thus, it should not be treated as separate type of value.

Alternative use value can also mean market value presuming that actual way of use would not be continued. EVS additionally explain that it is relevant to real estate, which value depends on business run on it.

4. Conclusions

There are many various types of value and therefore the valuer needs to present the definition of assessed value in transparent way not to mislead the valuation service's clients and to justify the chosen type value. Market value, as it is presented in this article, can significantly vary from other types of value. Every characterized value has its own specified rules of assessment and interpretation. Changing value base can result in crucial difference in valuation effect.

Definition of market value described in NBVS1 is consistent with general definitions written in international and European valuation standards and also with general definition appearing in EU regulations. It can be applied as a basic definition of market value for valuation purposes. However, in specific situations legal definitions or other regulations should be used. The most synonymous to market value is the term ‘fair value’ defined as accountancy term. Nevertheless, this term is applied in financial reporting to values other than market value.

It should be underlined, that while assessing market value or other than market value, the valuer has to apply relevant procedures and analyse sufficient amount of data to achieve reliable result. Market value is more relevant than other values to reflect real estate’s usefulness verified by market, while other values are created by characteristics specific for particular buyer or circumstances important for seller.

It should be also emphasized here that the purpose of professional standards is to unify the terms functioning in this wide theoretical and practical field as real estate valuation is. Among many assignments, the priority is to define the type of assessed value. It can be supported by the fact, that international, European and national standards including definitions explaining the types of value are treated as the base for valuation. All these documents include the clause that if the purposes of valuation or legal regulations demand accepting other definition of given type of value than written in above mentioned standards, the valuer should follow the rules.

In the date of writing this article the Commission for Standards PFVA was working over some modifications to the records defining market value.

1.2. INTERNATIONAL VALUATION STANDARDS AND NATIONAL VALUATION STANDARDS

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Key words: *real estate valuation, valuation standards*

Abstract

In the field of real estate valuation, standards were developed to unify the process of real estate valuation. They were established at different levels – first National Valuation Standards were introduced, and then European and International Valuation Standards followed.

The article proves that International Valuation Standards may not be considered an alternative to National Valuation Standards. It is not only due to the idea of IVS establishment, but also to the institutional dimension of the real estate market. Maintaining National Valuation Standards is also in the interest of Polish valuers.

1. Subject matter and purpose of the paper

The subject matter of this presentation is real estate valuation standards developed on the national level, hereinafter referred to as National Valuation Standards, and on the international level, hereinafter called International Valuation Standards. European Valuation Standards have been excluded from this analysis.

The purpose of this paper is to point out that International Valuation Standards must not be competitive with respect National ones. IVS are focused on processes and definitions, and do not include ethical rules which determine the conduct of property valuers in a given country or the complex technical components including accepted conditions for solving valuation problems. The latter is related to the institutional dimension of the real estate market.

2. Motivation for particular solutions

Despite their major internal differences, the circles of Polish valuation experts have been consolidated for years in a single body responsible for the preparation, arrangement, and application of professional standards.

The attitude of valuers to standards is extremely varied: some regard them as their professional Bible, while others – a limitation.

The latter group of experts put forth an initiative concerning their own professional organization whose members share the opinion that no National Standards should be developed, since it is perfectly sufficient to adapt

International Valuation Standards. The author believes that not only is this stance unfounded, but it may also prove detrimental to the process of valuation and to the whole community of property valuers.

3. Standards as a prerequisite for a separate profession

Every decision to follow a certain profession entails the acceptance of its approved conduct, systems of values, standards, and requires the acquisition of proper theoretical and practical knowledge, including the usage of notions understood in an unambiguous way. All this means that:

- one cannot describe a profession as a mere set of activities if there are no standards developed for them. For example: there is no such profession as "real estate market advisor", since no professional standards exist for this hypothetical profession. Therefore, being a real estate advisor is just a skill, not a profession;
- in order to follow a certain profession, one must observe particular standards developed for it.

The above observations show that the notion of "standard" is characterized by its wide terminological scope. If used as a noun, this term may refer to a set of behaviors, value systems, standards, definitions or procedures, while as an adjective it may refer to something typical, usual in terms of features or rules of conduct (procedures).

When we apply these considerations to the area of real estate valuation, we may observe the following:

- the development of valuation standards was a prerequisite to specify the profession of a property valuer;
- bearing in mind the ambiguity of the word "standard", the term "valuation standards" requires a further definition by specifying particular types of standards.

4. Types of valuation standards

The complexity of the valuation process and the significance of its results both for market players and the very functioning of the market require the co-application of four independent types of standards[1]:

- conduct standards;
- process standards;
- definitional standards;
- technical standards.

Conduct standards include a professional ethical code and the specification of required competences. In addition, they include minimum requirements for a document serving as the proof of an expert's work (i.e. an appraisal study). On the

[1] Sometimes called evaluation layers. See: P. Naubereit, The Harmonisation of Property Valuation – The Role of Valuation Parameters and Terminology, ERES 2008.

one hand, the rules of conduct are common for all professions, e.g. the requirement of professional honesty with respect to clients, of having proper qualifications, of not accepting tasks which exceed one's competences and possibilities, and on the other hand, they are specific and particularized, depending on the actual profession, by professional organizations and/or legislators. For instance, lawmaking bodies in co-operation with a professional organization specify the conditions of access to a profession, and requirements which have to be met by the representatives of a given profession in order not to lose their license or qualifications.

In countries with deeply-rooted professional organizations, these are those organizations which specify conduct standards. For instance, in Great Britain the standards in question for real-estate market players are established by RICS, in the USA, conduct standards for real-estate property management professionals are developed by IREM, also known and active in Poland, and for appraisers – by the Appraisal Foundation. The indicator of power for these organizations is the number of members, as well as their power of influence which greatly exceeds the borders of their countries of origin. These bodies have been growing in power for decades. RICS, which was established in 1863, and in 1881 obtained a Royal Charter, has become one of the major professional organizations in the world with approximately 100 thousand members from over 100 countries. RICS members work all around the globe, represented by national or regional branches in more than 50 countries. The conduct standards of this organization have been described in the so-called Red Book.

Another professional body, IREM, came into being in the interwar period as a reaction to problems posed by the management of real estate property taken over by banks in the era of the economic crisis.

In the countries without such well-established professional organizations, conduct standards have been greatly influenced by government regulations. Such a solution may be explained by their short history (e.g. in Poland, *Polska Federacja Stowarzyszeń Rzeczoznawców Majątkowych* or the Polish Federation of Valuers' Associations was established as late as in 1993), and on the other, put down to certain national features, for instance the German fondness of precise rules and order.

Process standards include a set of procedures which should be applied and observed by valuers undertaking a particular task. These procedures include the following:

- the requirement to specify the purpose and intended application of the valuation;
- the requirement to specify the date of the valuation and the appraisal study;
- the requirement to describe the subject of the valuation as both a physical and legal entity;
- a description of the underlying premises of the valuation;

- the indication of methods, techniques, and procedures which have led to the specification of a given market value;
- the indication of information sources;
- value level;
- limiting conditions;
- reservations.

Definitional standards are extremely significant for building a common language for evaluation. They are a tool to combine the theory and practice of valuation. These standards must cover the following fundamental notions: value, price, and cost. There was a proposition to define valuation parameters including the definitions of leasable area, the calculation of return rates, the calculation of income and expenditures, as well as data sources (NAUBEREIT 2008). In addition, definitional standards cannot be focused on definitions only, since they have to provide interpretations for particular categories. For instance, the definition of the market value of real estate or other property without a proper interpretation of individual words applied may lead to considerable variations in the understanding of the value, i.e. the very concept of market valuation.

Technical standards influence the valuation theory. For example, they include practical solutions for calculating income generated by properties of various functions (like an office block vs. a hotel), conditions for the acceptance of an model for an income generated by a property by means of applying certain valuation premises, the rules of converting income into value or the manner of calculating corrections for mapping the impact of real-estate differentiating factors on the value level of property under valuation.

All the above types of standards, although clearly set apart, do co-exist and interrelate. THORN (2007) is right in claiming that even an advanced valuation theory may prove poor in practice if it is presented by means of ambiguous and equivocal terminology. In other words, the theoretical structures applied must ensure the proper reflection of particular market conditions, and the very valuation must be an objective market image. Hence, professional ethics turns out to be quite useless if a valuer's opinion does not reflect the behavior of market players (THORN 2007). Therefore, THORN (2007) introduces the notion of "the standards matrix".

5. The origins and history of valuation standards

The history of valuation standards is relatively short. Despite the fact that people had been dealing with actual appraisal procedures for centuries, the first formal valuation standards appeared only in the second half of the 20th century.

The original attempt to establish such standards took place in Great Britain after the 1974 real-estate market crisis. A number of institutions began to examine the process of value calculation and question the credibility of appraisals. Thus, the Royal Institution of Chartered Surveyors (RICS) ordered to prepare a set of rules which was published in 1976 and has been known as The Red Book ever since. The Book was approved by the Bank of England, London Stock Exchange, London

Merger and Acquisition Committee, as well as by bank associations and other similar bodies.

In the USA, the first attempts to formulate valuation standards were taken after the Great Crash of the 1930s. Initially, they were based on codes of ethical conduct, and then sets of appraisal rules and good practices. However, uniform valuation standards were only introduced at the turn of the 1990s (DORCHESTER, VELLA 2000) by the Appraisal Foundation acting as an independent institution (BRETT).

These initial attempts to establish valuation standards were accepted as good valuation models in other countries. For instance, The Red Book was used as a prototype for value standards in a number of European, African, and Far-Eastern countries, whereas the American (USA) standards were accepted in Canada and Japan. However, although used as models, they were not directly applied by external users, due to many factors: habitual differences, distinct legal standards, history of the development of a given country, the level of market advancement, as well as access to market data conditioned not only by the activities within this market, but also by the development of organizations managing it.

Another strong stimulus for creating common and uniform valuation standards with an expanded geographical scope was capital flow. In 1981, the European Group of Valuers' Associations (TEGOVA) developed Europewide valuation standards, later referred to as The Blue Book. Initially, they were a copy of the Red Book, but later they became more and more influenced by the rules and regulations of the European Union (EDGE, 2000). The next edition of the Blue Book was entitled European Valuation Standards.

The desire to formulate common valuation rules was not limited to a single continent, since it was also manifested on the global scale (CHAMPNESS 1994). The leading role in this respect was taken by The International Asset Valuation Standards Committee (TIAVSC) established in 1981 and currently known as the International Valuation Standard's Committee (IVSC). The first edition of the International Valuation Standards appeared in 1985. Again, they were originally based on the notions and definitions taken over from the Red Book, but in 1994 they were rewritten to account for the needs and requirements of Northern American countries. Today, they are to serve as a reference point for users from all around the world.

Hence, the process of valuation standards development included the following three levels:

- the national level;
- the supranational level within a given continent;
- the international (global) level.

To sum up, the initial stimulus to develop the standards in question came from groups other than property valuers. Particular national standards served as a model for valuation rules prepared elsewhere.

A factor which intensified the process of standards harmonization was globalization. International capital flow, although it is not the only factor, has been playing a dominant role for the process of the globalization of estimations which must be comprehensible for investors, reflect common accounting rules **established on the international scale**, and must be clear for international creditors. The drive to establish uniform valuation rules also serves the interests of global real-estate advisory companies which cater to the needs of global business players and experts working in these companies.

The reasons for the unification of valuation rules on the international scale have not diminished. On the contrary, they have been and will be growing in force, due to, e.g. developments in indirect investment solutions in the real-estate market or the awareness of the fact that economic subjects influence the financial market by the manner of real-estate valuation (SAYCE, CONNELLAN 2003). However, the question is whether it means that only International Valuation Standards should be introduced on the national level and that National Valuation Standards have become completely obsolete?

6. Role and scope of International Valuation Standards

The establishment and popularization of IVS has at least three roles, i.e.:

– "to facilitate cross-border transactions and stimulate the recovery of international real-estate markets by promoting the rule of transparency in financial statements, as well as the rule of reliability of appraisals carried out for credit security, mortgage loans, property right transfer transactions, and tax settlements;
– to serve as a professional reference point or a signpost for property valuers all over the world to enable them to meet the requirements of international real-estate markets in terms of preparing credible appraisals, as well as the requirements of the world economic community concerning financial reporting;
– to provide valuation standards (…) which would satisfy the needs of emerging markets and newly industrialized markets" (IVS, 2005, p. 27).

The above means that apart from the harmonization of existing standards created by several bodies in different countries, IVS have a 'missionary' function due to the fact that they may be applied in developing countries.

Pursuant to THORN (2007), currently IVS are a mixture of various types of standards: definitional, process, conduct, and even technical ones, which influence the valuation theory.

7. Directions of changes within IVS

Nowadays, IVS are a mixture of rules, which makes them both a dictionary and a handbook (THORN 2007). A large group of influential observers have stated that in comparison with accounting standards, IVS in their current form do not provide a set of standards proper. The IVS Committee wants to focus on definitional and process standards, whereas the two other types, i.e. conduct and technical ones, are to be excluded from the scope of arrangements included in IVS, because:

– despite the fact that conduct standards make an essential component of valuation services, the Committee transfers the task of establishing the rules of ethical conduct to proper national organizations (professional bodies or institutions empowered to issue professional qualifications). The IVS Committee is not a professional institution, and neither does it have legislative powers;
– the Committee believes that imposing valuations rules on particular countries would be regarded as a threat, would limit the development of valuation methodology which should be free in order to be successfully improved in reply to evolution both in market and academic thought (THORN 2007).

The above exclusion of valuation methodology issues from the scope of IVS is strongly supported by British valuers, since they are of the opinion that methodologies as such are dynamic phenomena which change in reaction to needs, fashion, demand, and analytical techniques. Some believe that methodology should be but a component described in handbooks, whereas any set of tools has a practical form regulated by national standards, and others that methodology should be included into national standards as their integral part (NAUBEREIT 2008).

Since its establishment, the IVS Committee was cautious in terms of including valuation methodology issues into the standards. This attitude was manifested, for instance, in the lack of standards concerning the calculation of real-estate value, income generation or the establishment of rules for adjustments in order to reveal the influence of a real-estate differentiating features on the value of valuated property.

The Committee created a working group including the representatives of valuation and accounting experts in order to:
– establish the format of an optimum set of international valuation standards, with their structure, content, and scope;
– review the latest, eighth edition of IVS (2007) in terms of its strong and weak points, and to compare it to the version deemed as optimum;
– prepare a roadmap for further work over IVS development (IVS 2007, typescript).

To sum up, despite the fact that the two latest editions of IVS (the seventh from 2005 and the eighth from 2007) do include references to conduct standards (code of conduct) and technical standards (application of International Valuation Standards and the Interpretation Guidelines), we may assume that they will be largely limited in next editions to definitional and process ones (THORN 2007). In addition, we may assume that the level of IVS' generality will be further increased.

8. National Valuation Standards as a means to particularize International Valuation Standards

The previous considerations indicate that International Valuation Standards leave the scope of regulations for national ones. The difference between the two lies in the fact that International Valuation Standards may be applied everywhere, whereas national ones either expand or modify the valuation rules. Thus, they are adjusted to the actual needs of particular countries (THORN 2007). The above may be supported by the following statements which prove that the IVS Committee:

- "… aims at creating a situation to make IVS and particular national standards reciprocally complimentary and supportive" (IVS 2005, p. 28); and realizes the fact that
- "Local legal regulations and economic conditions may in some cases require special (and sometimes limited) application of … IVS" (IVS, 2005, p. 37).

The global rules in this respect remain largely generalized. "Instead of explaining the application of certain procedures or methodologies, they rather provide a recommendation as to what valuers should do" (IVS 2005, p. 28). The procedures and methodologies should be explained in National Valuation Standards and applied in the valuation process by national valuers. Pursuant to the Committee's intention, IVS are not meant as competition to National Standards. NVS are supposed to confirm and provide additional details to the general rules. The IVS Committee states as follows "… national organizations of property valuers should take every effort to make new national standards reflect best valuation practices included in IVS" (IVS, 2005, p. 290).

IVS must not be an alternative to NVS. In the area of general definitions and rules, NVS should evolve towards IVS. They should provide details for the general rules in question by discussing valuation methods and adjusting them to the level of development of a particular national market which includes:

- property rights;
- codes of conduct;
- professional organizations;
- social attitudes;
- tradition;
- practice.

The above components create the so-called institutional aspect of the market, which conditions the acquisition and acceptance of valuation standards, and consequently the very valuation process and real-estate value level. The market aspect in question may largely limit the freedom of penetration and adaptation of the international standards of valuation of real estate as an asset.

9. Potential consequences of replacement of NVS with IVS

The author believes that the fact of accepting IVS in lieu of NVS would entail a number of negative consequences, because:

– the high level of generality of IVS gives room to various interpretations, e.g. within the area of income calculation, establishing the rates of return, the conversion of income into value or copying legal solutions concerning property right limitations;
– the replacement in question would lead to increasing differences in the value level of the same real estate calculated by different valuers at the same time and for the same purpose. Not only would it be conditioned by different interpretations of acquired data, valuation assumptions (being the objective source of the differences), but also by diverging interpretations of valuation methodology and sources of data acquisition by particular experts. All of this would increase the so-called uncertainty of valuation, which in turn would undermine the reliability of both valuation reliability and property valuers' work, along with their professional kudos.

10. Conclusions

The process of globalization has increased the demand for global estimates, for instance for the needs of financial statements prepared by business entities with international capital. This situation required the creation of a common, global valuation language, along with global procedures. These requirements may be well met by IVS which, apart from the much-needed unification of definitions and the valuation process, have generally started to provide rules of conduct for property valuers and explanations of valuation methodology. The two latter fields may be largely limited in the future, as they may be included in the scope of arrangements at the national level.

Valuation standards are undoubtedly essential to all market players, not only their recipients, but also property valuers, since they enable them to define and precise the product that these experts offer, and establish the required level of qualifications, which is a prerequisite for creating a full-fledged professional environment that could establish proper prices for the services offered. The standards in question provide a kind of protection against potential competition (THORN 2007).

Not only do uniform national regulations decrease the professional risk and increase the prestige by making the rules more detailed, but they also indispensible to the creation of a healthy valuation services sector within the national market. And despite the fact that some valuers may regard these standards as burdensome shackles, the very existence and observance of these rules serves the interests of valuers, although not every valuation expert working in international markets knows how to use them properly.

1.3. QUALITATIVE AND QUANTITATIVE METHODS FOR ASSESSING THE SIMILARITY OF REAL ESTATE[2]

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Key words: *multi-dimensional modelling of the real estate market, similarity criteria, correlation, qualitative methods, quantitative methods*

Abstract

This paper proposes using qualitative and quantitative methods to establish similarity criteria of real estate. The problem of assessing the similarity is a key issue in the comparative approach to real estate appraisal in which the appraiser selects properties most similar to the appraised one from a collected database.

Algorithms for establishing the degree of similarity may be based on estimating absolute differences between qualitative characteristics, for example using one of the following methods: relative similarity analysis, property ranking analysis.

To combine qualitative and quantitative methods, weights of particular property characteristics in their prices can be included in the above algorithms for selecting similar properties. These weights are determined for example based on the coefficients of correlation between the attribute and the property price. Thus, when performing the relative comparison analysis or the ranking analysis, we take into account only significant attributes of properties selected based on their weights. The correlation coefficients used can be determined based on the ranks assigned to qualitative variables or on the numerical values of these variables resulting from their scaling, which gives them a qualitative character. The correlation coefficients proposed for use are: Pearson's correlation, Spearman's correlation, Kendall's correlation, Gamma correlation.

This paper presents examples of using various methods of property similarity assessment and evaluates their impact on the final result of the appraisal made using a defined algorithm.

1. Introduction

The assessment of similarity between a real estate to be estimated and these to be the basis for the assessment is the first and principal stage of a comparative approach of real estate estimation. A skilled expert, knowing the local market can choose, without any special algorithms, among many real estates under

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transaction, these which are near to the subject of estimation in respect of essential features. However, considering the importance of choosing objects similar to the estimated one – they are the basis for the whole further assessment process in a comparative approach – it is proper to examine the influence of similarity estimation method on the final effect of assessment i.e. on the real estate value as well as on the accuracy of evaluation. This means the application of a concrete mathematical algorithm.

2. Two-stage algorithm of real estate assessment

The starting point for analyses carried out on a local market of a determined type of real estate is a numerous set of market data concerning transactions concluded within the close vicinity of the assessment date. On the basis of these data, the assessment model parameters are estimated, in the form of a multi-dimensional function, verified in view of matching to the market data as well as in view of its parameters stability and, in consequence, the reliability of the prediction established on this basis. Real estate evaluation using the estimated multi-dimensional model is done in conjunction with a full accuracy analysis.

The basis for the model value prediction done in such a way is the whole database, which, in the case of multidimensional function models, has to be numerous enough and, in consequence, it is frequently much diversified. Hence, in the second prediction stage, the value obtained directly from the model is corrected by the correction calculated on the basis of a small group of real estates most similar to the estimated one, selected from the input database. Detailed description of the two-stage algorithm for determining the final market value of real estate is presented in the work (BARAŃSKA 2007b).

2.1. First stage of real estate assessment – multidimensional modeling

2.1.1. Forms of estimated assessment models

Models, which parameters were estimated in the first stage are:

- linear additive model (multiple regression)

$$c = \sum_{i=1}^{m} \left(x_i - \hat{x}_i \right) \cdot a_i \tag{1}$$

where:

c – *real estate unit price,*
x_i – *value of attribute* i,
$\hat{x}_i$ – *mean value of attribute* i *in database,*
m – *number of considered attributes,*
a_i – *model parameters.*

- non-linear additive model

$$c = \sum_{i=1}^{m} \left(a_{i1} \cdot (x_i - \hat{x}_i) + a_{i2} \cdot (x_i - \hat{x}_i)^2 + ... + a_{in_i} \cdot (x_i - \hat{x}_i)^{n_i} \right) \quad (2a)$$

or in a more general form

$$c = \sum_{i=1}^{m} f_i \left(x_i \right) \quad (2b)$$

where:

n_i – *degree of the polynomial for i-th attribute, describing the dependence of real estate price on this attribute,*

$f_i\,(x_i)$ – *function of price dependence on attribute* i,

other notation like in formula (1)

The assessment of parameters in models mentioned above is performed according to the algorithms described, among others, in the works Barańska (2007) and Barańska (2008) using the least squares method.

2.2. Methods of selecting similar real estates

In this work, four different methods (criteria) were used for assessing similarity between real estates:

– analysis of real estate relative comparison,
– analysis of scheduling (ranking) real estates,
– selecting similar real estates by their essential attributes,
– selecting similar by three attributes decisive for the price.

Two first methods consider all real estate attributes as their equivalent features creating the prices. They are based on algorithms described with examples of using in study CZAJA AND PARZYCH (2007). Two other methods take into account only these attributes, which were, in a special selection, positively verified in view of their significant effect on the variability of real estate prices within a given type.

2.2.1. Analysis of real estate relative comparison

The analysis of a relative comparison is based on a quality comparison of similar real estate attributes with the attributes of the estimated real estate, aiming only to catch differences, without considering their magnitude. For each feature of the compared real estate, a correction is set up in form of +1 or -1, depending on that if it is worse or better than the corresponding feature of the estimated real estate. The sum of corrections gives a total correction positive or negative. It means that the compared real estate has summary attributes correspondingly "worse" or "better". Real estates with a summary correction equal to zero or the cheapest from the "better" ones and the most expensive from the "worse" ones are recognized as the most similar to the estimated real estate.

2.2.2. Analysis of real estate ranking

It is essential in this method to set up the position for each compared real estate as well as for the estimated real estate within the whole database, in relation to the theoretical real estate, to which we assign the smallest (or the greatest) values of all attributes. These ranges result from the summary number of corrections for individual attributes. However, as opposed to the analysis of relative comparison, assigning a correction in consideration of a given attribute, we take into account of how many degrees of its scale it differs from the reference extreme value. In this way, the corrections take the values of integers with the same signs, for example 0, -1, -2 and so on, when the reference point is the "worst" real estate and 0, +1, +2 and so on, when the reference point is the "best" real estate.

Real estates with the ranges such assigned are arranged in a determined hierarchy. As the most similar to the estimated real estate we consider these occupying the same position or eventually two adjoining real estates, i.e. the cheapest with "better" of one range and the most expensive with "worse" of one range.

2.2.3. Selecting similar real estates considering their essential attributes

In the algorithm of similarity assessment considering essential attributes, the key to estimating is to select among all features describing real estates within a database – the ones, which create their prices in a most essential way. To select such features, different correlation coefficients were used:

- Pearson's correlation,
- Spearman's correlation,
- Kendall's correlation
- Gamma correlation.

As is well known, Pearson's correlation coefficient examines only the degree of linear relationship between random variables. For the reason that for its calculating prime values of random variables are used – it is greatly sensitive to the occurrence of conspicuous cases. The remaining correlation coefficients, as a correlation range – are resistant to the conspicuous cases, but they have their restrictions as well, capturing only the monotonic relationship between variables. Regarding this, all these correlation types were used to the variables in estimated models after carrying out the process of the model linearization. Thereby, transformed data were achieved, for which it was justified to choose explanatory variables, essential for creating an explanatory variable (price) on the grounds of weight shares determined by the correlations mentioned above. At the same time, using such procedure can reveal in every model, even on the same market, other features significantly influencing the price.

After selecting from among a dozen or so of real estate features several most important, the information on which was gathered at the stage of creating the database, the choice of real estates similar to the estimated one is done, for example, by applying one of the methods described above (p. 2.2.1 or 2.2.2) only in relation to the selected attributes.

2.2.4. Selection of similar real estates on the basis of three attributes mostly influencing the price

The procedure of estimating similarity on the basis of three real estate features most important for the price creation is done as for the algorithm discussed in 2.2.3. However – on the basis of weight shares – we constrain more the number of features constituting the criterion for selecting similar real estates.

Having only three features of a real estate, which we consider as important, we do not need any more for a similarity assessment to apply one of the methods described in 2.2.1 or 2.2.2. The selection of similar real estates can be done by selecting from the database the elements having the same scale values for three selected attributes.

2.3. Second stage of real estate assessment – correcting the model value

Estimated parameters of the assessment model allow determining the systematic component of the model for database unit prices of real estates, being the base for estimation. These are the values of real estate price predictions obtained from the model, i.e. model values W. Every systematic component includes a collection of model random deviations δ, with inaccuracy characteristic contained in its covariance matrix:

$$[\delta]=[C]-[W]=[C]-[X]\cdot[X]^{+}\cdot[C]=\left[I-X\cdot X^{+}\right]\cdot[C] \tag{3}$$

$$Cov[\delta]=\hat{\sigma}_0^2\cdot\left[I-X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\right] \tag{4}$$

where:

$[W]$ - vector of real estate model prices,

$[C]$ - vector of real estate actual prices in a database,

$[\delta]$ - random deviations of assessment model.

X – coefficient matrix of equation system in matrix form, created for the model after the linearization,

X^{+} – *pseudo-inverse of matrix* X,

$\hat{\sigma}_0^2$ – *estimator of remainder variance.*

From the systematic model, considering the values of the estimated real estate attributes, we assess its model market value w_M, and applying the law of variance escalation, we determine the standard deviation, describing the accuracy of this assessment $\sigma(w_M)$.

From the vector of random component [δ] we isolate deviations δ_{i_w}, corresponding to the *k* selected real estates, the most similar to the estimated one, and from the random deviation covariance matrix – the submatrix $Cov[\delta_w]$ sized $(k\times k)$, containing the elements corresponding to the selected deviations.

According to the procedure described in BARAŃSKA (2007b), from a random model we assess the market value of a random correction to the model value for the estimated real estate w_L and its standard deviation $\sigma(w_L)$:

$$w_L = \left[\underline{1} \cdot P \cdot \underline{1}^T \right]^{-1} \cdot \left[\underline{1} \cdot P \right] \cdot \left[\delta_w \right] \tag{5}$$

$$\sigma^2(w_L) = \sigma^2_{0_w} \cdot \left[\underline{1} \cdot P \cdot \underline{1}^T \right]^{-1} \tag{6}$$

where: $\underline{1} = [1 \quad 1 \quad \ldots \quad 1]$ – vector composed of ones, sized $(1 \times k)$,

$P = Cov^{-1}[\delta_w]$ – weight matrix being covariance matrix inverse,

$\sigma^2_{0_w}$ – remainder variance determined for a group k of selected real estates.

The final prediction of the estimated real estate market value is the sum of the systematic component and the determined random correction. As the accuracy of the model value and random correction is known, the final result is given also with the accuracy analysis.

3. Subject of analyses

Market information on two different types of real estate, coming from three different local markets, was subject to the analysis. Information concerned the transactions concluded during maximum of 20 months.

The first two databases contain data on land unbuilt real estate, destined for farm or apartment building with the possibility of service, situated on the grounds of the communes Zabierzów and Wielka Wieś within Cracow district. Ground features were described using the following attributes: area, zone, mode, mpzp, road, legal restrictions, territorial development, arduous environment elements, destination of adjoining grounds, plot shape, topography.

The third database contained dwellings from the towns Wadowice and Andrychów situated at a distance of 15 km. Preliminary analyses permitted to admit that the markets of both towns are alike, mainly in view of the level of dwelling market prices. Yet, undoubtedly, Wadowice as the town of John Paul II and as the district seat is more attractive as regards tourism and has larger potential of economic development. An additive premise to join information from both towns in one database was the number of market information. To perform a reliable estimation of parameters of multi-dimensional function models, it is necessary to have a sufficient number of data. This was confirmed among others by analyses done on many databases in the works [BARAŃSKA 2003, 2004]. The following attributes of dwellings were considered: communication access, position of the dwelling in the building, building technology, mode, building condition, surroundings, and distance from the objects of public use, balcony or loggia presence, zone.

Three created databases contained 76 to 93 cases described above. In every database, one of the real estates from among gathered data was subject to the estimation, so its price obtained as result of concluded transaction was known. To predict real estate model value only these models were used, which demonstrated, on a given market, sufficiently good matching to the market data and passed successfully through a statistic verification process, described in detail in papers BARAŃSKA (2005 and 2007a).

4. Variants of modeling real estate unit value

At the stage of estimating function model parameters mentioned in 2.1, we analyzed separately the case, where the date of transaction was treated as one of the attributes, i.e. one of the explanatory variables and separately the case, where the prices were corrected before modeling, accordingly to the explanation presented by the paper author [Adamczewski 2006]. In second variant, the form of the function representing variability of prices in time on a given real estate market was chosen, as best it was possible, and in process of modeling real estate values, prices transformated to one point on time axis, i.e. to the estimation date, were used. Further calculations have been done in parallel for both variants, as we failed to select functions correcting the prices since the time of transactions showing good matching to the data was different. Curvilinear correlation coefficient was within 0,36 to 0,59.

Additionally, both databases of ground real estates showed high diversification, i.e. low data coherence. Regarding this and in view of their large amount – an additional calculation variant was considered for data reduced by about 50% (41 of 93 real estates from Zabierzów and 38 of 76 from Wielka Wieś), but decidedly more coherent and from the very beginning more close to the estimated real estate.

4.1. Selection of essential attributes and similar real estates

It appeared that there is no significant differences in applying different correlation coefficients do selecting the most important real estate features influencing their prices in all databases.

In the cases where a chosen method of selecting similar real estates gave the same results as a previous one – tables 1 and 2 below contain final predictions of real estate values only for one of them.

Table 1 contains the results of selecting similar real estates using different methods, from among described in 2.2. On the basis of data presented in the table we can see that the effect of selecting similar real estates depends on the method used for it. Only in three separated/distinguished cases, 100% of real estates considered as the most similar in a method were recognized as similar using another method and only one of these cases applies to the situation, where all methods selected, in practice, the same real estates as the most similar to the estimated one.

Table 1

Number of real estates considered as the most similar

Method of selecting similar real estates		Analysis of relative comparison	Analysis of ranking	Attributes mostly influencing the prices	Three attributes mostly influencing the prices	Number of real estates repeating in all methods
Additive linear model						
Zabierzów n=93	transactional	7	6	9	4	2
	corrected	11	2	13	8	2
Zabierzów n=41	transactional	3	3	5	–	1
	corrected	9	6	–	–	**6**
Wielka Wieś n=76	transactional	14	9	5	4	3
	corrected	23	12	35	–	**12**
Wielka Wieś n=38	transactional	4	5	7	–	2
	corrected	12	11	–	4	0
Wadowice, Andrychów n=61	transactional	7	3	4	–	1
Additive non-linear models						
Zabierzów n=93	transactional	4	3	3	–	1
	corrected	9	6	8	–	5
Zabierzów n=41	transactional	3	3	4	–	2
	corrected	3	3	11	–	0
Wielka Wieś n=76	transactional	8	5	16	–	1
	corrected	12	4	9	–	1
Wielka Wieś n=38	transactional	4	3	5	–	0
	corrected	6	4	9	–	0
Wadowice, Andrychów n=66	corrected	9	9	8	–	**8**

Source: own study

5. Results of market value prediction

The purpose of the applied two-stage model of real estate estimation is to state precisely a real estate value determined on the basis of well fitting to the local market and statistically verified function estimation model. Its advantage is to distinguish in a database of similar real estates a group of real estates marked by the greatest similarity to the estimated object and use them to "correct" the real estate model value. Table 2 contains the results of a two-stage modeling of market values of ground real estates and dwellings using different multi-dimensional models of function forms (1)-(2), after considering the corrections determined on the basis of random deviations of real estates most similar to the estimated one, selected by different methods.

Table 2

Final predictions of real estate values

Linear additive model – multiple regression									
Method of selecting similar real estates		Analysis of relative comparison [zł/m2]		Analysis of ranking [zł/m2]		Attributes mostly influencing the prices [zł/m2]		3 najbardziej cenotwórcze atrybuty [zł/m2]	
Baza		W_{M+L}	$\sigma(W_{M+L})$	W_{M+L}	$\sigma(W_{M+L})$	W_{M+L}	$\sigma(W_{M+L})$	W_{M+L}	$\sigma(W_{M+L})$
Zabierzów n=93	trans.	167,47	7,38	174,26	7,78	170,51	6,35	183,78	5,14
	correc.	160,01	5,08	161,83	6,68	157,23	4,67	158,27	5,34
Zabierzów n=41	trans.	172,86	3,31	170,96	2,88	167,06	3,35	–	–
	correc.	129,69	3,72	128,00	4,35	–	–	–	–
Wielka Wieś n=76	trans.	121,37	5,55	121,98	6,30	124,37	8,48	111,58	4,75
	correc.	148,26	5,31	152,41	7,14	150,66	5,03	–	–
Wielka Wieś n=38	trans.	108,57	2,22	110,45	2,04	110,28	2,02	–	–
	correc.	135,96	2,75	135,43	2,90	–	–	139,36	3,54
Wadowice	trans.	2328,50	99,72	2317,46	96,98	2328,92	85,73	–	–
Non-linear additive models									
Method of selecting similar real estates		Analysis of relative comparison [zł/m2]		Analysis of ranking [zł/m2]		Attributes mostly influencing the prices [zł/m2]		3 najbardziej cenotwórcze atrybuty [zł/m2]	
Baza		W_{M+L}	$\sigma(W_{M+L})$	W_{M+L}	$\sigma(W_{M+L})$	W_{M+L}	$\sigma(W_{M+L})$	W_{M+L}	$\sigma(W_{M+L})$
Zabierzów n=93	trans.	195,56	11,18	196,54	5,52	209,32	9,62	–	–
	correc.	179,60	6,19	186,86	6,05	183,35	5,53	–	–
Zabierzów n=41	trans.	174,01	3,12	173,69	3,06	174,74	2,67	–	–
	correc.	130,45	2,92	128,91	2,57	130,93	2,40	–	–
Wielka Wieś n=76	trans.	95,43	7,17	96,51	7,22	94,76	5,92	–	–
	correc.	131,61	7,56	134,76	7,36	131,87	7,96	–	–
Wielka Wieś n=38	trans.	110,31	2,82	109,30	2,59	108,06	2,08	–	–
	correc.	151,09	1,71	150,07	2,00	151,81	1,58	–	–
Wadowice	correc.	3530,05	68,74	3511,00	57,52	3518,95	67,52	–	–

Source: own study

6. Comparison of prediction results for different method of selecting the most similar real estates

The results listed in table 2 are presented with standard deviations of final predictions of real estate values. To carry out the inference on the effect of using a concrete method of selecting real estates the most similar to the estimated one on the final result of the value prediction in a two-stage model of real estate estimation, several statistical tests have been performed. The aim of the tests was to compare both the predictions themselves and their accuracy.

The form of test function in the test comparing real estate values:

$$Z = \frac{W_1 - W_2}{\sqrt{\sigma^2(W_1) + \sigma^2(W_2)}} \tag{7}$$

The form of the test function in the test comparing predictions accuracy:

$$F = \frac{\sigma^2(W_1)}{\sigma^2(W_2)} \tag{8}$$

Table 3 contains only these results of parametric tests performed on the significance level of 0.05, for which test function values indicate statistical significance of verified difference. In the first column of each table, a symbolic description of compared pairs of considered cases is used with the notation as follows:

D, M – bases: numerous (large) and reduced (small),

T, K – prices: transactional and corrected,

L, A – models: additive linear, additive non-linear,

1–4 – consecutive methods of selecting similar real estates described in 2.2.

Table 3

Results of tests comparing final predictions of real estate values and their accuracy

Compared pair	W_{M+L} (1)	$\sigma^2(W_{M,+L})$ (1)	W_{M+L} (2)	$\sigma^2(W_{M+L})$ (2)	Z_{obl}	Z_{tab}	F_{obl}	F_{tab}
Ground real estate from Zabierzów								
1-4 DTL	167,47	54,40	183,78	26,37	-1,81	1,96	**2,06**	1,51
2-4 DTL	174,26	60,48	183,78	26,37	-1,02		**2,29**	
3-4 DTL	170,51	40,29	183,78	26,37	-1,63		**1,53**	
1-2 DKL	160,02	25,85	161,83	44,56	-0,22		**1,72**	1,53
2-3 DKL	161,83	44,56	157,23	21,81	0,56		**2,04**	
2-4 DKL	161,83	44,56	158,27	28,56	0,42		**1,56**	
1-2 DTA	195,56	125,09	196,54	30,48	-0,08		**4,10**	1,52
2-3 DTA	196,54	30,48	209,32	92,56	-1,15		**3,04**	
Ground real estate from Wielkiej Wsi								
1-3 DTL	121,37	30,82	124,37	71,96	-0,30	1,96	**2,34**	1,60
2-3 DTL	121,98	39,68	124,37	71,96	-0,23		**1,81**	
2-4 DTL	121,98	39,68	111,58	22,57	1,32		**1,76**	
3-4 DTL	124,37	71,96	111,58	22,57	1,32		**3,19**	
1-2 DKL	148,26	28,21	152,40	50,99	-0,47		**1,81**	1,60
2-3 DKL	152,40	50,99	150,65	25,30	0,20		**2,02**	
Dwelling								
1-2 TL	2328,50	237,47	2317,46	776,18	0,35	1,96	**3,27**	1,56
1-3 TL	2328,50	237,47	2328,92	2835,56	0,01		**11,94**	
2-3 TL	2317,46	776,18	2328,92	2835,56	0,19		**3,65**	
1-2 KA	3530,05	1233,41	3511,00	2651,22	0,30		**2,15**	1,57
2-3 KA	3511,00	2651,22	3518,95	1401,00	0,12		**1,89**	

Source: own study

On the basis of the results shown in table 3 we can see that applying different methods of selecting the most similar real estates influences only the accuracy of the final prediction of a real estate value. There is no evidence that this affects the height of the prediction itself. Following regularities were observed:

- For the data from the **commune of Zabierzów**, the method of selecting similar real estates considering up to 3 attributes most important for the additive linear model, based on the transaction prices, at a large database, provides the best accuracy of the final estimation of the value.
- The analysis of ranking real estates gives the lowest accuracy of final estimation for the additive linear model, based on the prices corrected, at a large database.
- The analysis of ranking real estates provides significantly better accuracy than the analysis of relative comparison and selection of similar on the ground of the attributes most influencing the prices, for the additive non-linear model, estimated for transaction prices in a large database.
- For the data from the **commune of Wielka Wieś**, the method of selecting similar real estates on the ground of all most important attributes for the additive linear model, based on transaction prices and on a large database, gave the lowest accuracy of the final prediction of the estimated real estate value. Also, on the list of methods giving the lowest accuracy, the method of ranking real estates is on the second place. The most precise prediction was achieved by assessing similarity of real estates by considering their three most important features at the maximum.
- The model of multiple regression based on corrected prices and estimated in a large database shows as well a low accuracy of the analysis of ranking real estates.
- For the data from **Wadowice and Andrychów**, all additive models (linear and non-linear) indicate that the analysis of ranking real estates and selecting similar on the ground all essential attributes are the methods giving significantly lower accuracy of the final prediction than the analysis of relative comparison.

7. Summary

The paper presents different methods of selecting similar real estates and the effect of applying them on the prediction of a real estate value as well as its accuracy in a two-stage estimation algorithm. Statistical tests permit to deduce that the method of selecting real estates most similar to the estimated one does not significantly influence the final prediction of the value, but it has often a significant effect on the accuracy of the prediction. There are no evident regularities, though the values with the largest standard deviation occurred mostly after applying the analysis of real estate ranking as a method of selecting the most similar. However, the assessment of similarity only on the basis of three most important attributes often provides the most precise prediction of the value. It seems then that in the process of real estate assessment there is no necessity for considering a large number of features while we can choose the most significant among them. A great number of explanatory variables do not influence significantly the final value, though it has a disadvantageous effect on its accuracy.

CHAPTER 2

THE PROPERTY MANAGEMENT PERSPECTIVE

2.1. MODEL OF INTERACTIONS BETWEEN MUNICIPALITIES AND THE REAL ESTATE MARKET

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Key words: *real estate market, municipality, land management, modeling*

Abstract

As managers of vast land resources, municipalities are in the possession of tools and instruments that enable them to influence the real estate market, mostly indirectly. This interaction can be presented by models of relations and dependencies which provide information on processes that stimulate local growth. This study discusses the key principles of descriptive modeling of market phenomena, and it attempts to build a simplified model of interactions between the municipalities and the real estate market. The significance of the Olsztyn municipality and the role it plays on the local land market are also analyzed.

1. Modeling as reality simplification

A model is a simplified, hypothetical structure that maps reality in a simplified way and describes its most characteristic features, phenomena, conditions or processes. It may exemplify a specific type of behavior, conduct or practical activity, and reflect the dependencies between actions and results (KOGUT-JAWORSKA 2008). The concept of a model is used in various contexts and it may have different interpretations. In research and experimental procedures described in literary sources, the term "model" is used for two main purposes:

- identifying a theory that structurally resembles another theory, thus enabling a smooth transfer from one theory to another by simply changing the applied terminology; in this sense, a model is an instrument of cognition;
- identifying a system to which a given practical concept or a theory applies to simplify the description of the investigated natural system; in this context, the model is an object of cognition (ZEIGLER 1984).

In general terms, modeling supports the investigation of a given process which is replaced with a simplified representation displaying only selected attributes of the investigated phenomenon. The nature and degree of simplification are determined by the researcher's level of knowledge, needs and awareness, and it may change subject to the objective of the study. Both a theory and a model perceive reality in a simplified, abstract form. The complexity of the key determinants and processes observed on the real estate market and the dependency between the activities initiated by the municipalities, which play an important role on that market, and the results of those efforts give a new significance to the modeling of such interactions. The interactions between municipalities and the real estate market are modeled to describe, explain and forecast market phenomena (Fig. 1). The descriptive part should focus on the municipality's activities and the situation on the real estate market, the explanatory part involves a search for cause-and-effect relations, while forecasting implies using the knowledge of the mutual interactions and complexities to assess the municipality's efforts. As regards the municipalities' activities and their contribution to local development, a model should be build with the aim to:

- provide a relatively comprehensive overview of the mechanisms governing the real estate market against the backdrop of the local economy,
- formulate a cohesive description of real estate management processes,
- map reality in view of changes taking place in the surrounding environment.

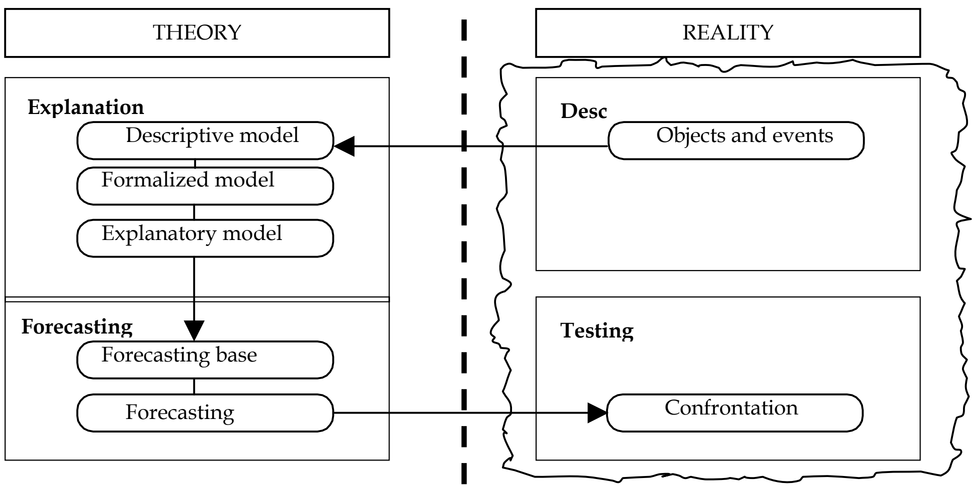

Fig. 1. General principles of descriptive modeling. *Source*: Own study based on JASZCZYK AND SZCZERBICKI (2003)

The concept of a model is interpreted in narrow, econometric terms in analyses of economic phenomena, in particular prices on the real estate market. In this case, an economic phenomenon is described by a formalized model with the use of mathematical statistics and econometrics.

The classification or typology of model types is developed in many ways to produce, for example, analogical, mental, mechanical or descriptive models (JASZCZYK, SZCZERBICKI 2003). Subject to the purpose of modeling, there are structural models which illustrate the selected attributes of a given phenomenon and the relations between those attributes, as well as functional models that show the way in which selected attributes and relations affect the analyzed processes.

From the point of view of the model's nature, models can be divided into diagnostic, heuristic and virtual. Diagnostic models include monitoring, regulatory and test models. Heuristic models can be subdivided into intuitive, questionnaire and expert models, while virtual models cover iconic, logical and mathematical models. Diagnostic models represent reality with the use of observation, control and measuring tools (field research). As regards heuristic models, the knowledge is acquired by indirectly examining an existing object (desk research). Virtual models mirror reality by investigating virtual objects, such as a computer simulator (POWIERŻA 1977). Diagnostic, heuristic and virtual models may be applied for the needs of real estate management. The anticipated results and the conclusions from model analysis support the evaluation of the real estate market as well as spatial decisions which are directly or indirectly related to the local real estate market.

2. Interactions between municipalities and the real estate market

A municipality is a local body representing the public authority. It is a separate self-governing unit which is independent from other public sector entities and which may actively participate in market trade. In most cases, the objective of the local development policies of a municipality is to support entrepreneurship and attract capital. The exerting of influence on the real estate market is not a goal in itself, but an element of local policy which relies on legal, economic and financial instruments in the real estate segment. The involvement of the municipality, a market entity, in that process may be one of the many factors that stimulate economic growth by correctly identifying the needs of other market actors and undertaking suitable measures to effectively meet those needs.

The effectiveness of the measures undertaken by municipalities on the real estate market is highly contingent on the effectiveness of financial and real estate management practices. The interactions between the municipalities' activities and the real estate market should be analyzed not only from the perspective of the local demand, supply and prices, but also in the broader context of their contribution to local economic growth.

The model illustrating the municipalities' contribution to local growth is based on various elements characteristic of macroeconomic models, models of coexistence with the public sector and models shaping interactions at the local level (local development models). Special attention should be paid to models that illustrate the stimulation of economic growth based on the municipality's assets. In those models, a municipality's economic potential is shaped through the implementation of local development policies. A model illustrating the concept of economic growth stimulation based on the municipality's assets is presented in Fig. 2.

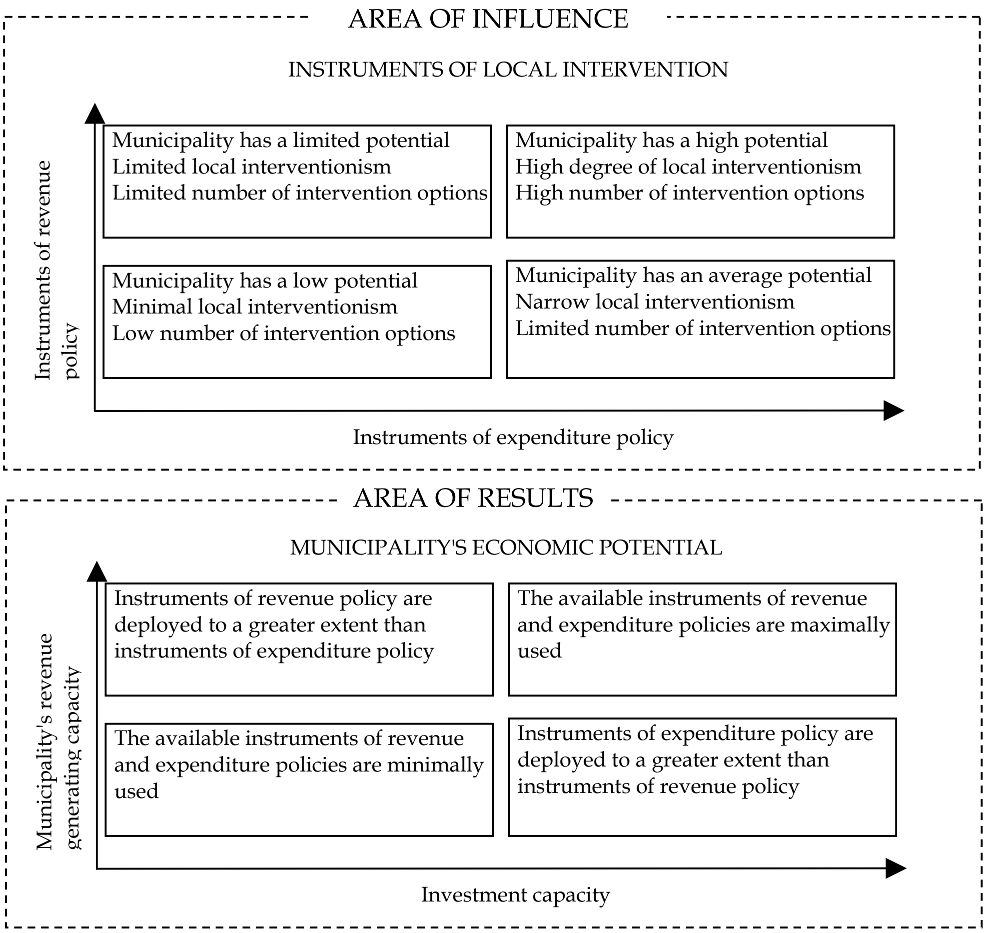

Fig. 2. Model illustrating the concept of economic growth stimulation based on the municipality's assets. *Source*: Own study based on KOGUT-JAWORSKA (2008)

Models of local economic growth stimulation should supply information for the improvement of the quality and effectiveness of the self-government's activities. The aim of this model should be to provide information concerning the following processes:

– adapting the structure of the economy to local expectations,
– adapting the social and economic system to a free market reality,
– autonomous processes that reflect the internal needs of the local economic system,
– expanding the structures of the local economic system in a long-term perspective (KOGUT-JAWORSKA 2008).

The influence exerted by the municipalities on the real estate market is most clearly manifested in real estate management. The main group of instruments deployed in real estate management include strategic and operational planning instruments, implementing instruments comprising legal and administrative measures as well as economic and financial instruments (TOPCZEWSKA, SIEMIŃSKI, 2003). Municipalities perform the following detailed tasks as part of real estate management:

- enacting local laws,
- adopting resolutions in cases relating to property rights,
- selling and granting perpetual usufruct rights to land,
- setting the terms and conditions for real estate use by organizations,
- setting rates and charges for the lease of municipal real estate,
- determining the manner and the dates for land development,
- pronouncing opinions on and approving the division of real estate,
- merging and dividing real estate,
- taking advantage of pre-emption,
- developing technical infrastructure and acquiring land for development projects,
- real estate tax assessment and collection.

As an important actor on the local real estate market, the municipality's influence is felt in all areas that serve the public and the local community. The municipality's operations on the real estate market are governed by a set of rigorous legal procedures. The most important legal regulations determining the terms and the scope of the actions undertaken by large-scale proprietors on the real estate market are: general legal provisions, regulations that authorize a party to become involved in civil law transactions on the real estate market, regulations determining the terms of real estate management, regulations determining the terms of housing resources management, regulations determining real estate protection and real estate tax assessment. The main legal act which regulates the central and the local authorities' participation in real estate market transactions in Poland is the Act on Real Estate Management. Non-tender sales procedures have significant implications for the real estate market, in particular as regards the sale of municipal housing to the existing tenants. Perpetual usufruct procedures also have important consequences for the real estate market as the price of the property may significantly differ from its market value.

The municipalities' activities and the results of those efforts on the real estate market may be analyzed in a spatial, legal, economic and social context as part of a larger system (Fig. 3).

In most cases, the municipalities affect the real estate market through a set of indirect measures. It should be noted that municipalities are often in the possession of highly specific types of real estate (areas zoned for road construction, barren land, areas built up with technical infrastructure). This property can be put up for sale, but even more importantly, it has a vast potential which, if adequately deployed by an individual or a commercial buyer, it can lead to significant changes in the spatial structure, the transport network, the municipality's image, and, consequently, the market value of the surrounding areas. For this reason, the municipalities' related efforts should be perceived as an element of a larger system where the consequences of those efforts, i.e. transaction prices, are a derivative of the municipalities' intervention in different areas of social and business life.

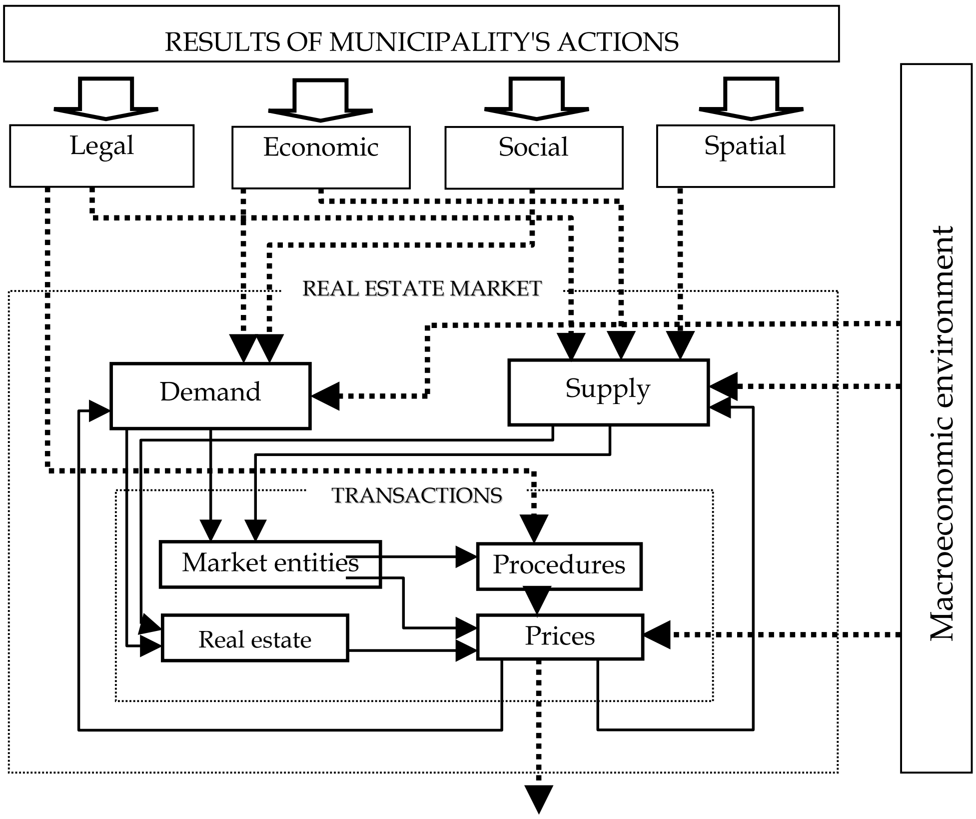

Fig. 3. Simplified model of the municipalities' interactions with the local real estate market. *Source*: Own study.

For every system to be adequately described by a model, the connections between the identified elements and the existing dependencies have to be determined. From the analytical point of view, the preferred solution would be to express those relations in mathematical form. This is not always possible, however, even if abundant data are available. The majority of characteristic market attributes are expressed in semantic form which, combined with low levels of information, may disable the use of mathematical tools. The above prompts the search for other, sometimes not tested, solutions. In general, the model can be expressed as follows:

$$M \leftrightarrow \{D, R, W, P\}$$

where:

D – the model domain in terms of observation points or isolated systems,

R – relations within the model domain,
W – results and conclusions from model analysis,
P – algorithms developed based on processed market data.

The model domain may comprise the municipalities' legal and economic instruments as well as objects that constitute both material (e.g. property, market participants) and intangible assets (e.g. property rights, time, terms of transaction), but those categories are usually immeasurable in the physical sense. The domain of a model encompasses relations relating to the municipalities' activities initiated with the use of various tools as well as the ensuing results, i.e. changes in the objects' attributes.

3. The municipality's share of the real estate market in Olsztyn

The Olsztyn municipality has a total 41% share in ownership rights and perpetual usufruct rights to land situated within the boundaries of the city of Olsztyn (Table 1). As regards the number of registered land plots, this approximation delivers less objective results because the number of land plots is not directly proportional to land area. The municipality's share of the market clearly indicates that the manner in which the local authorities manage their real estate resources significantly affects land supply and, consequently, the prices of land.

Table 1

Land assets of large-scale proprietors in the city of Olsztyn

<table>
<tr><th rowspan="2">Legal title</th><th colspan="2">Olsztyn municipality</th><th colspan="2">State Treasury</th><th colspan="2">Military Property Agency</th><th colspan="2">University of Warmia and Mazury</th></tr>
<tr><th>Area [ha]</th><th>No. of plots</th><th>Area [ha]</th><th>No. of plots</th><th>Area [ha]</th><th>No. of plots</th><th>Area [ha]</th><th>No. of plots</th></tr>
<tr><td>ownership</td><td>3014.7</td><td>6385</td><td>82.4</td><td>162</td><td>919.6</td><td>293</td><td>491.8</td><td>504</td></tr>
<tr><td>perpetual usufruct</td><td>629.4</td><td>5359</td><td>728.3</td><td>1502</td><td>-</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>%share in ownership</td><td>34.12</td><td>21.29</td><td>093</td><td>0.56</td><td rowspan="2">10.41</td><td rowspan="2">1.01</td><td rowspan="2">5.57</td><td rowspan="2">1.73</td></tr>
<tr><td>% total share</td><td>41.26</td><td>40.32</td><td>9.18</td><td>5.71</td></tr>
</table>

Source: Own study.

According to the author's findings, the municipality participates in approximately 26% of real estate transactions on the local market as the buyer or seller of undeveloped land. Transactions involving legal entities other than the municipality have a 38% share, while transactions between individuals account for 36% of all real estate transactions in the city of Olsztyn. The number of transactions for the sale/purchase of undeveloped land in each year, with a division into transaction types, is presented in Fig. 4.

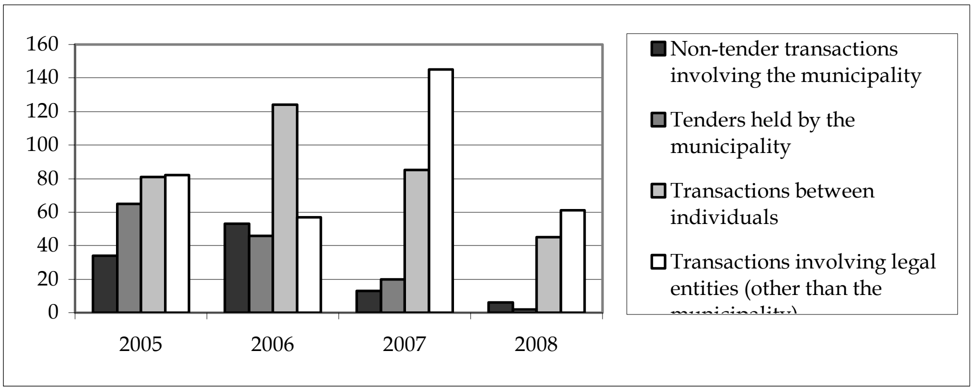

Fig. 4. Number of transactions for the sale/purchase of land in Olsztyn with a division into transaction types. *Source*: Own study.

The number of transactions is not always a reliable indicator of market performance, because the investment capacity of market actors can only be determined based on the total area of the sold land. The data on the total sold land area presented in Figure 5 indicate that legal entities have a relatively high market share. They are mostly developers who buy land for residential construction.

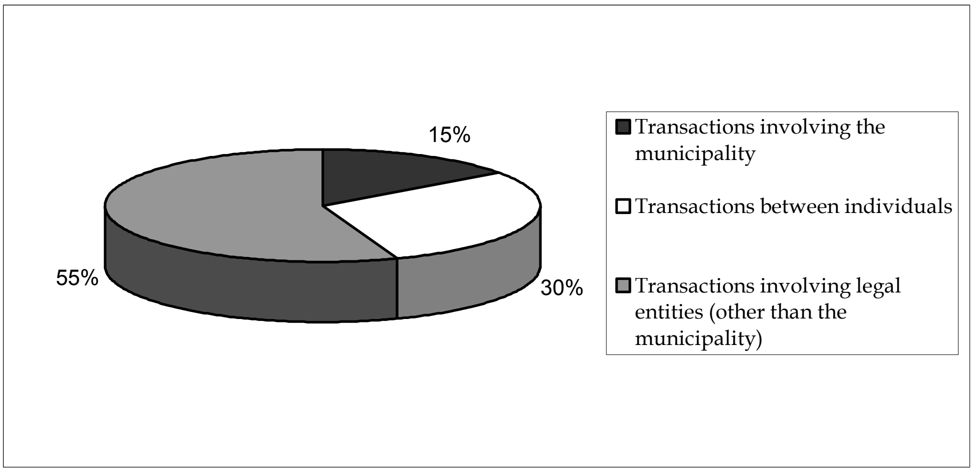

Fig. 5. The share of the municipality, individuals and legal entities in the total area of undeveloped land traded on the Olsztyn market. *Source*: Own study.

A market analysis revealed that the prices noted in transactions between individuals were lower than the prices quoted during tenders where the municipality acted as the selling party. As regards non-tender transactions in which the municipality sold land for the purpose of improving utility development standards in the area, the quoted prices were lower than in transactions between individuals (Fig. 6).

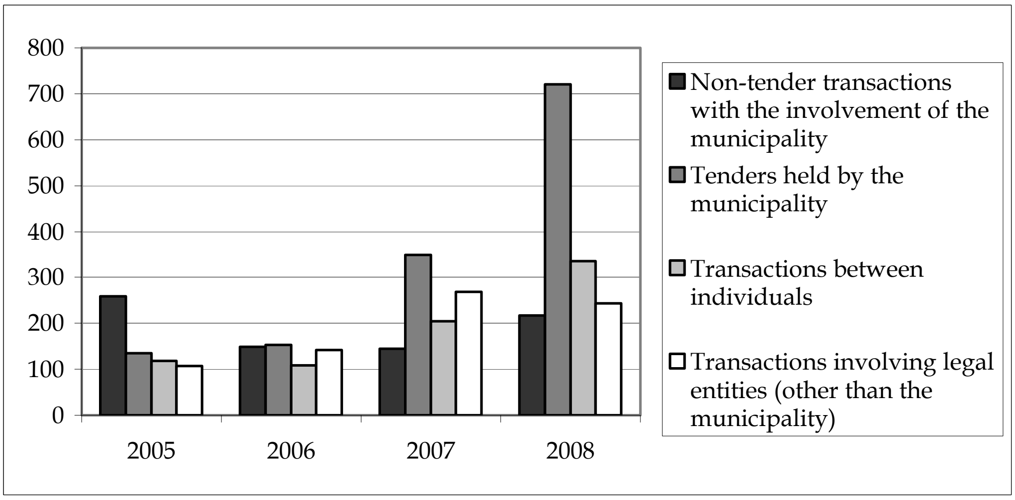

Fig. 6. Average unit prices of land with a division into transaction types. *Source*: Own study.

A growing trend is reported in particular as regards undeveloped land sold by the Olsztyn municipality by way of a tender. Yet due to the low number of land plots sold during tenders in 2008, the average quoted price is not representative for this type of transactions. The average was substantially raised by a transaction in which a piece of land with a prime location was sold for nearly PLN 900 per m^2.

4. Conclusions

The modeling of interactions between the municipalities and the local real estate market is a valuable tool which serves an evaluation purpose, but primarily it supports the initiation of rational actions to stimulate local growth. Real estate, in particular land, is the most valuable natural asset of every city and municipality. By nature, land assets are not limitless, they are gradually depleted and this fact may stilt local development. As managers of vast land resources, municipalities are in the possession of tools and instruments that enable them to influence the real estate market. By controlling the supply of real estate, municipalities influence the value and the market prices of land assets. The municipalities can also set the direction for local growth by initiating own development projects and offering attractively located and priced land plots to external investors. Territorial governments are in the possession of various financial instruments for controlling demand, including taxes, charges and special economic zones. They also have at their disposal technical and planning instruments for modifying the zoning designations, merging and dividing real estate. The municipalities' policies may, therefore, initiate a variety of positive changes on selected segments of the local market.

2.2. ANALYSIS OF THE TENDENCY OF COMMUNES TO USING THE POSSIBILITY OF ALTERNATIVE SHAPING OF THE PRINCIPLES OF CARRYING IN FEES FOR THE PERPETUAL USUFRUCT OF GROUNDS

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Keywords: *perpetual usufruct, real estates, tendencies*

Abstract

In the article applying of elements of the theory regarding the tendency for the attitude research of communes within the scope of using mechanisms of shaping principles of paying a fee for the perpetual usufruct in an alternative way in relation to standard statutory principles was presented. Results of the questionnaire survey conducted on communes of Zachodniopomorski district were discussed.

1. Introduction

The perpetual usufruct is a form of ruling national and self-government real estates by natural and legal persons raising controversy almost from the moment of familiarizing in 1961 (Kokot 2003). Although this institution was repeatedly modified, in 1990 in a way adapted to principles of market economy (Źróbek 2003), these controversies aren't weakening. Certainly one of the main causes is appointing by opinion-forming circles (politicians, journalists) a negative image, deduced from socialist origins of this institution. However, it is a non-metoric cause and the institution of the perpetual usufruct functioned in practice without serious reservations, that kind of arguments wouldn't defend themselves. One should seek fundamental reasons of the recalled crisis in other spheres, and an imperfect economic structure of the perpetual usufruct should be found among them undoubtedly. One should determine this structure as stiff. Systems making up the economical construction of the usufruct:

- first fees,
- annual fees,
- payments for the failure to adhere to dates of developing the real estate,
- payable transfer of the perpetual usufruct into the ownership of the real estate,
- sale of real estate for perpetual user,
- discounts from payments connected with the perpetual usufruct.

Some of mentioned systems have a complementary character like e.g. systems of payments and the system of discounts.

To the stiffness, straight out causticity character of conditions, on which real estates are being given back into the perpetual usufruct the most important influence have factors like:

- amounts of rates percentage of annual fees determined by law,
- dense connection of the amount of coupon rates of annual fees with the aim of giving back the real estate to perpetual usufruct,
- dates of contributing annual fees indicated by law.

A set of by law determined principles was established in terms of each of each system of the economical structure of the perpetual usufruct. In reference to part of them, these principles have the same character as standard principles with the possibility of modifying them in a defined scope by applying mechanism indicated by law. It means that to a certain extent, the individual giving real estates back into the perpetual usufruct has an opportunity to form these principles in an alternative way to the one indicated in the act of standard management of real estates. Four mechanisms of liberalizing principles of paying a fee for the perpetual usufruct are to distinguish:

I. applying increased rates of annual fees for the perpetual usufruct (act 76, sec. 1),

II. applying discounts from fees for the perpetual usufruct due to entering the real estate in a list of vintage buildings in other than standard (50 %) height (act 73, sec. 4

III. applying discounts from fees for the perpetual usufruct in situations mentioned act 68 sec. 1, pt 1 - 6, 8 and 9 acts of the management of real estates (art. 73, sec. 3), i.e. for the real estate allocated for indicated purposes or sold for specified entities

IV. allowing for contributing annual fees by perpetual users in other time than till 31 March (act 71, sec. 4).

2. Research methods

In the article findings of communes concerning the tendency of using these possibilities were described. Perforce so-called tendencies of exogenous character were the object of examination, i.e. these formed by extrinsic factors such as: the legal system, the knowledge and experience of people at commune offices responsible for matters regarding the management of real estates, opinions of other persons whom persons responsible for matters of the management of real estates are contacting and exchanging views, effects of media factors, as well as personal experience of the clerks.

A frequency method of K Popper (1996) is one of the essential methods of measurement of the tendency. This method is identifying the measure of the tendency with the fraction of events in which the examined phenomenon appeared with reference to the number of the attempt. So a relative frequency of the incident of the given phenomenon is:

$$p(x) = \frac{m}{n} \tag{1}$$

where:
m - number of incidents of the given phenomenon,
n - number of all possible events.

This measure was chosen for examinations introduced in the work hereinafter, on account of its simplicity, legibility and the easiness of interpretation (Hozer and Doszyń 2004). For greater legibility of results, the measure of the tendency was expressed in per cents. Examinations were conducted in 2007 in form of a questionnaire forms on the population of communities of Zachodniopomorskie district, including the following questions (among others):

1. whether the local government applies increased rates of annual fees for the perpetual usufruct,
2. whether the local government applies discounts from fees for the perpetual usufruct due to the entry of the real estate in a list of vintage buildings in other than standard (50 %) height,
3. whether the local government applies discounts from fees for the perpetual usufruct in situations which are talked about in art. 68 sec. 1, pt 1 - 6, 8 and 9 acts of the management of real estates (these are optional discounts, which the commune can, but does not have to apply specified situations),
4. whether the local government allows for carrying annual fees by perpetual users in other time than till 31 March,

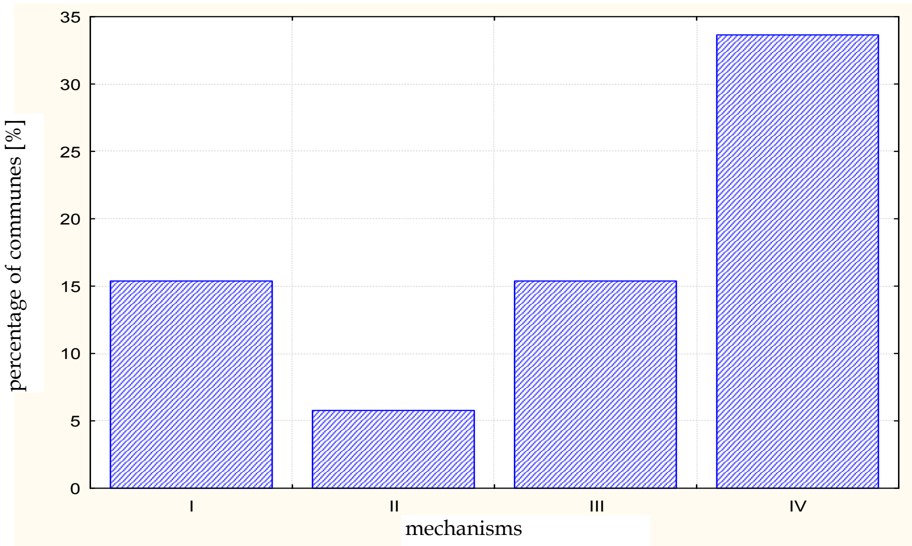
Managers of departments dealing at commune offices with matters of the economy regarding real estates were asked for responses, being given the possibility to make a choice from one of two replies:

- yes,
- not.

3. Results

Results were drawn up considering the categorization of communes in respect to:

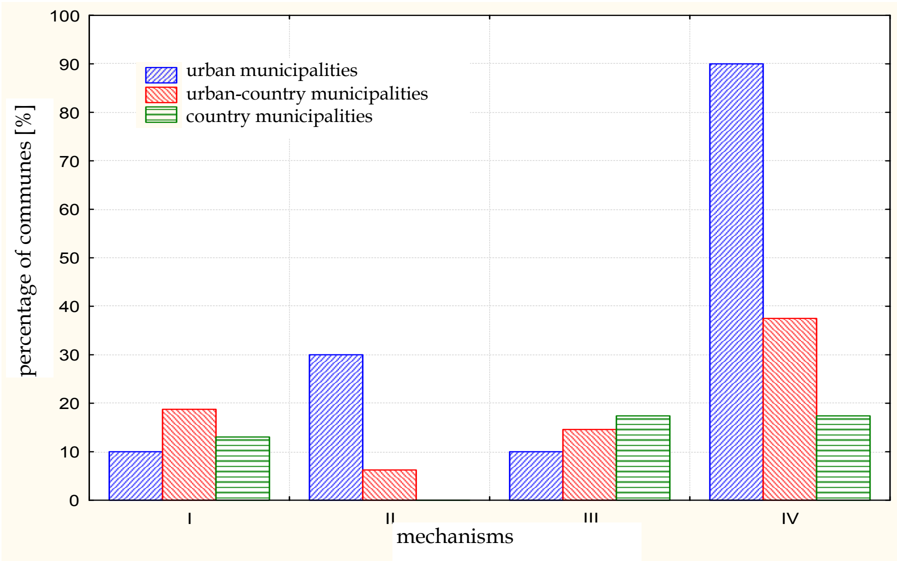
- kind (urban, urban-country and country),
- participation of ground in the perpetual usufruct in ground constituting the property of the commune (to 10 % and above 10 %),
- population (to 10,000 and above 10,000),
- own income of the commune to 1 inhabitant (up to 1,000 zloty and above 1,000 zloty)
- number of entities of the national economy to 1000 inhabitants (to 100 and above 100)



Graph 1. Tendency of communes to use mechanisms of alternative shaping principles of paying a fee for the perpetual usufruct of ground. *Source*: own study.

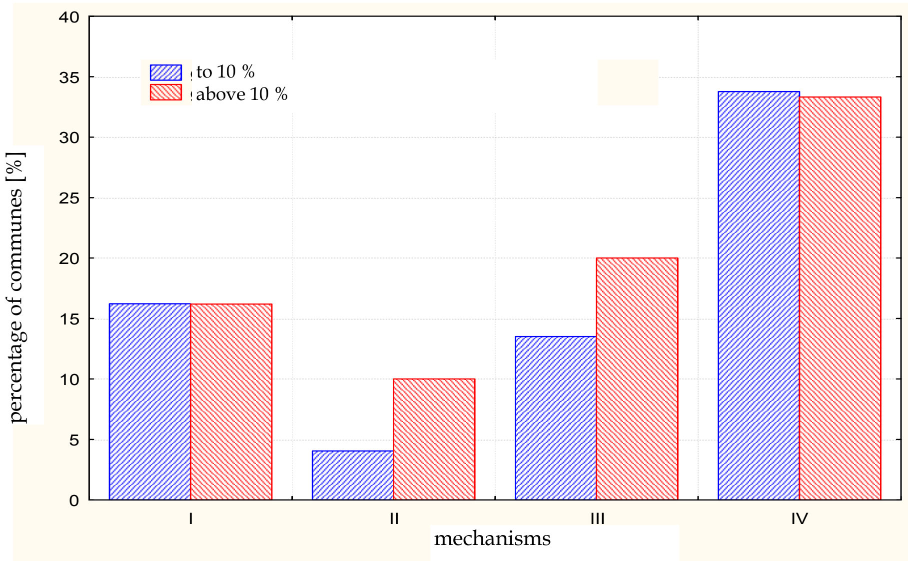
As can be seen on graph 1, the tendency of communes to apply mechanisms of liberalizing the principles of paying a fee for the perpetual usufruct is generally rather small. The greatest tendency was observed for the IV mechanism, i.e. for applying by communes the possibility of carrying annual fees in other way than on a one-off basis till 31 March every year. But also here the tendency gained only 34 %. Scarcely 15 % of the communes are declaring to apply mechanisms I and III, i.e. applying increased rates of annual fees and applying discounts from payments in specified situations. Definitely communes are demonstrating the lowest tendency for applying the mechanism no II, i.e. of applying the discount from fees for the perpetual usufruct in relation to writing down the real estate in a list of vintage buildings in other height than 50 %. To this tendency, however, a relative thinness of appearing of the monuments devoted to the real estate into the perpetual usufruct and simultaneously entered in a register is overlapping each other. Low tendency of communes to apply specified mechanisms for provokes the question why. One should seek for the answer in two technical areas of principles of functioning of communes. Either statutory mechanisms are so good, that a legitimacy of their alteration arises rarely or the majority of communes is demonstrating the specific ambivalence and indolence in the attitude to these elements of the economy of real estates, and maybe also for the management of real estates in general.

On the next graph a tendency of communes using discussed mechanisms was described in division into urban, urban-country and country municipalities.



Graph 2. Tendency of communes to use mechanisms of alternative shaping of the principles regarding paying a fee for the perpetual usufruct of ground in the division into urban, urban-country and country municipalities. *Source*: own study.

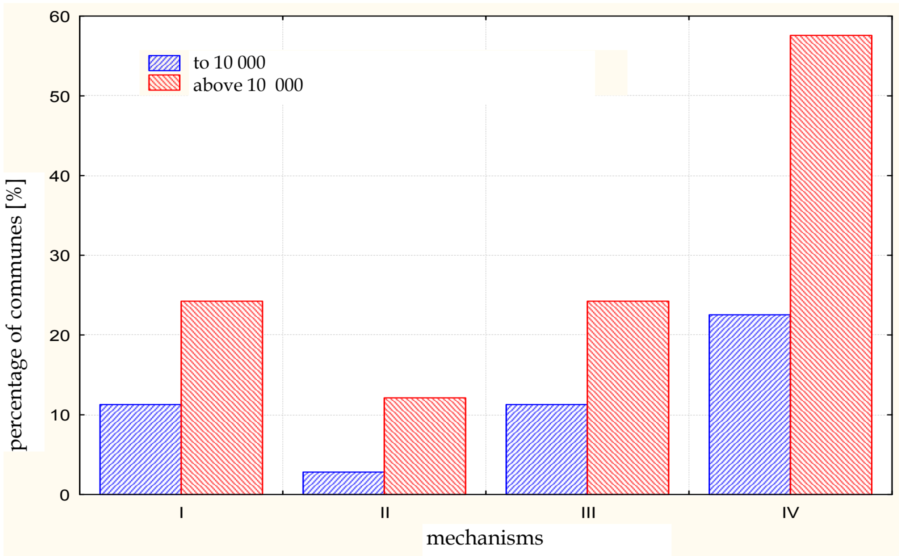
Describing findings of the tendency of communes to apply discussed mechanisms in the division into categories is indicating great diversity of the communes in this field. While e.g. municipalities are demonstrating 90 % - for making contributing annual fees more flexible, for mixed communes this tendency is gaining tendency level of 38 %, and for rural communes only 18 %. The tendency for the alteration of the amount of the discount in relation to the entry of the real estate to a list of vintage buildings for municipalities is 30 %, and rural communes aren't using this mechanism generally speaking. Rural communes have biggest (18 %) tendency to apply discounts on base of act 73, sec. 3, and paradoxically it is the smallest tendency for municipalities (10 %). There occurs a strange tendency to apply the mechanism I, where mixed communes are dominating. However, one should emphasize that differences in the level of the tendency for mechanisms I and III are relatively rather small and one can think that properties of the examined community of communes affected such a forming.



Graph 3. Tendency of communes to use mechanisms of alternative shaping principles of paying a fee for the perpetual usufruct of ground in the division into communes surrendered into the perpetual usufruct to 10 % and above 10 %. *Source*: own study.

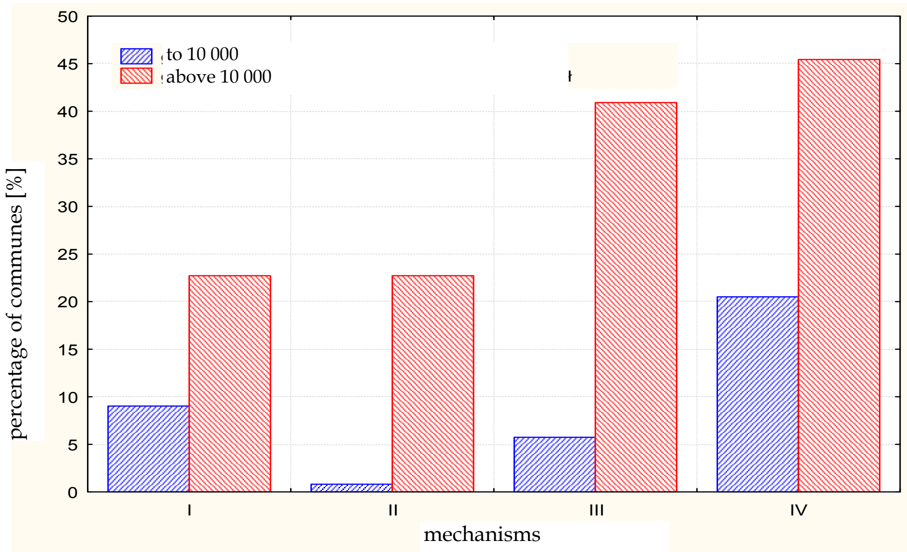
One should conclude from graph 3 that percentage of ground the local government deposited into the perpetual usufruct has no indicative influence to the level of tendency to apply discussed mechanisms of liberalizing principles of paying a fee for the perpetual usufruct. Peculiarly clearly one can see the lack of these differences for mechanisms I and IV, where tendencies for applying them irrespective to the percentage being to 10 % or above 10 % is practically the same. However, for II and III mechanisms, i.e. mechanisms concerning applying discounts, they are a little bit (about 6 - 7 %) higher for communes of greater participation.

Totally different levels of discussed tendencies are to be observed depending on the size of the commune measured with the number of inhabitants in the division into communes to 10 000 and above 10 000 of inhabitants (graph 4). Although, this criterion is in a way correlated with the division of communities into urban, urban-country and country, results received here are not identical with results presented on graph 2. In each of distinguished mechanisms it is possible to notice the clearly greater tendency to apply them for communes with the number of inhabitants above 10 000, from 9 % for the II mechanism, to 35 % for the IV mechanism.



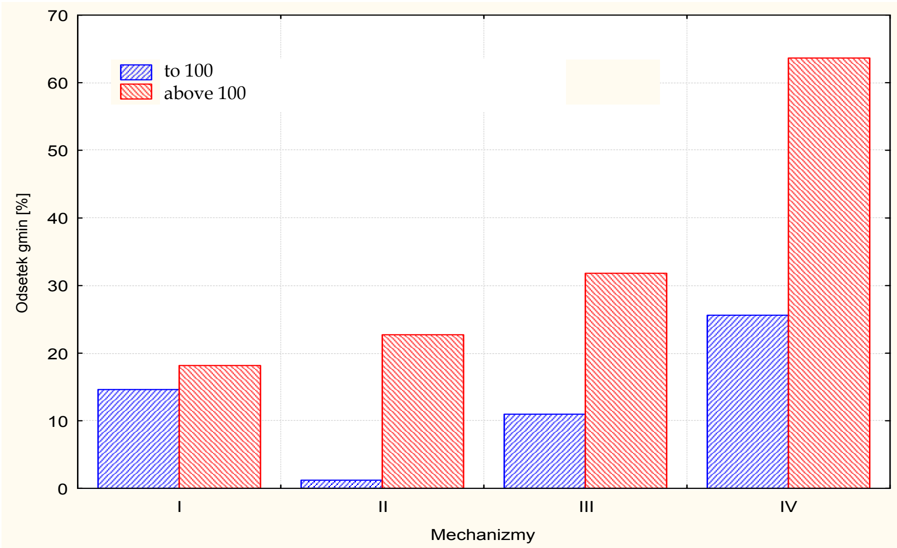
Graph 4. Tendency of communes to use alternative sharing mechanisms principles of paying a fee for the perpetual usufruct of ground in the division into communes in terms of the number of inhabitants to 10 000 and above 10 000. *Source*: own study.

More visible differences of presented tendencies are still seen in the division of communities on account of the level of the own income of communes expressed for 1 inhabitant of the commune (graph 5). Communes with own income above 1 000 zloty for 1 inhabitant are demonstrating a significantly greater tendency to apply each of four discussed mechanisms. These differences are smallest for the mechanism I, where are equal to 15 %, and biggest for the III mechanism – 35 %. For II and IV mechanisms they are also over 20 %. It proves existence of connection between the tendency for shaping relations of the commune with perpetual users of real estates into the, probably matching flexible manner for market needs and the level of the economic development of communes. It is possible on this example to attempt to put forward the thesis that the economic development of the commune is in the certain measure associated with the style of leading by her the management of real estates in broader meaning, because communes are showing similar elasticity like in the mechanisms associated with the perpetual usufruct also in other spheres of the management of real estates conducted by them.



Graph 5. Tendency of communes to use mechanisms of alternative shaping principles of paying a fee for the perpetual usufruct of ground in the division into communes in terms of own income for 1 inhabitant to 10 000 zloty and above 10 000 zloty. *Source*: own study.

Similar, very distinct differences in the level of the tendency of using mechanisms of liberalizing principles of paying a fee for the perpetual usufruct are seen at the division of communities considering the number of subjects for 1000 inhabitants (graph 6). These tendencies, especially with reference to II, III, and IV mechanisms are definitely higher for communes with the number of entities to 1000 inhabitants bigger than 100. Differences in the level of measures of the tendency, in comparison to communes with number of subjects below 100 to 1000 inhabitants are for II and III mechanisms 10 %, and for the IV mechanism – 20 %. Bearing in mind the fact that the number of entities in proportion to the number of inhabitants can be treated as the measure of the social-economic development communes, achieved results are confirming the connection between the tendency of communes to as far as possible elastic formulation of principles of cooperation and the relation with its inhabitants and entities being functional in its area, and the level of its development. It seems that higher level of development of the commune is resulting in such attitude of the local government towards the accomplishment of statutory tasks, however, as mentioned previously proving such a thesis requires carrying much more research. It isn't possible also to rule out on the basis of results presented in the present article, that it is inversely, i.e. higher level of development of the commune kind of is forcing it into applying custom mechanisms in specific situations.



Graph 6. Tendency of communes to use mechanisms of alternative shaping of principles of paying a fee for the perpetual usufruct of ground in the division into communes with number of entities equal to 1000 inhabitants to 100 and above 100.
Source: own study.

4. Conclusions

In table 1 number of communities using the determined amount of specified mechanisms was described. It turns out, that half of communes (52) which took part in the examination is using none of them, and only one local government applies all 4 mechanisms.

Table 1

Amounts of mechanisms of making principles of paying a fee for the practiced perpetual usufruct by communes more flexible

No of mechanisms	Number of communes
0	52
1	36
2	12
3	3
4	1

Source: own study

In table 2 are indicated the coefficients of Person's linear correlation between frequencies of applying discussed mechanisms, which is supposed to answer a question, whether a link between applying specified mechanisms exists. It turns out that rates have minimum values and only applying the II mechanism indeed is correlated with applying the III mechanism and applying the III mechanism indeed is correlated with applying the IV mechanism (on the p importance level below 0.05), at least and in these connections coefficients of correlation aren't high.

Table 2

Coefficients of Person's linear correlation between applying mechanisms of liberalizing principles of paying a fee for the perpetual usufruct applied by communes

Mechanizm	I	II	III	IV
I	1,00			
II	0,12	1,00		
III	0,04	**0,24**	1,00	
IV	0,09	0,09	**0,20**	1,00

Source: own study

Analysis of achieved results causes the following general conclusions. Communes are demonstrating a low tendency of using instruments making statutory solutions more flexible such as applying custom rates of annual fees or granting optional discounts from payments, from different reasons predicted by law, as well as allowing for contributing annual fees in other times than till 31 March. Although the tendencies are rather small one can observe apparently clearly outlined diversifying in this respect regarding specified criteria. These tendencies are clearly lower in rural communes, with small population numbers, with low own income to one inhabitant and with the low figure of subjects to 1000 inhabitants. In many cases differences of measures of the tendency for communes qualified for all sorts groups are very significant and they reach the multiple. Relatively, the lowest diversity in this regard was made on account of the criterion of the participation of ground in the perpetual usufruct in ground constituting the property of the commune, at least and in this case this tendency was for two mechanisms greater for communes with higher participation of ground in the perpetual usufruct. It leads to a general conclusion that larger and richer communities treat the institution of the perpetual usufruct at run by them managements of real estates in a conscious and sensible way more appropriate to needs.

In the context of the described examination a question arises, whether the socio-economic development of the commune is associated with the style of leading by it the entirety of the management of real estates? Probably similar elasticity like in the sphere of the perpetual usufruct is shown by the communes in other fields of managements of real estates. The answer to that question requires, however, conduction of separately designed research.

CHAPTER 3

MATHEMATICS IN REAL ESTATE MARKET

3.1. TIME SERIES OF RESIDENTIAL PROPERTY PRICES IN POLAND - IDENTIFICATION OF NONSTATIONARITY TYPE

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Key words: *time series of residential property prices, nonstationarity type, unit root test*

Abstract

In recent years, the residential property market in Poland has undergone dynamic change which resulted in a steep increase in property prices. These changes have specific implications for the time series of residential property prices. Data-generating processes in the property market system are conditioned by different causal factors which result in the absence of stationarity and stability in the time series of residential property prices in Poland. A nonstationary stochastic process is a process where the probability distribution function varies over time. The time series of residential property prices may be characterised by nonstationarity in average and nonstationarity in variance. The observed nonstationarity is not homogeneous. This study presents the results of an analysis to identify the nonstationarity type in time series of residential property prices on the Polish market.

1. Stationarity of time series of property prices

Stochastic processes can be divided into stationary and nonstationary processes.
A stochastic process $c(t)$ is stationary when it is not characterised by time correlations and when variables have identical probability distribution. In practice, this means that a process is stationary if its probability distribution function remains constant in reference to displacement over time (MANTEGNA STANLEY 2001).
An asymptotically stationary stochastic process takes place when the distribution of random variables $c(t_1+x),\ldots,c(t_n+x)$ is not determined by x for large x (MANTEGNA STANLEY 2001). A stochastic process with stationarity of N-th order takes place when probability density meets the following equality:

$$f(c_1,c_2,\ldots,c_n;t_1,t_2,\ldots,t_n)=f(c_1,c_2,\ldots,c_n;t_1+x,t_2+x,\ldots,t_n+x) \quad (1.1)$$

not for all n, but only for $n \le N$.
A stochastic process is nonstationary if presented condition (1.1) is not met. Based on this assumption, PIŁATOWSKA (2003, p. 26) differentiates the following nonstationary processes:

1) processes which are nonstationary in average and stationary in variance,
2) processes which are nonstationary in variance and stationary in average,

The nonstationarity of a stochastic process is related to the nonstationary characteristics of its first-order normalised moment and second-order central moment. The nonstationarity of stochastic processes is discussed at length by PIŁATOWSKA (2003).

The nonstationarity of time series of property prices is considered to be a serious problem in statistical analyses (cf. CHAREMZA DEADMAN 1997). Nonstationarity often has to be eliminated in economic applications, in particular in analyses of dependencies between nonstationary processes. Nonstationarity is eliminated to bring the entire analysis to dependencies at the level of stationary processes. The attempt to analyse dependencies at the level of stationary processes is dictated by the fact that the vast majority of statistical methods are based on the assumption of process stationarity. The most frequently cited example of doubtful results is the application of regressive models in the analysis of nonstationarity in time series of prices. The problem of "apparent dependency" has been researched for many years (cf. GRANGER NEWBOLD 1986; PHILLIPS 1986; CHAREMZA DEADMAN 1997). Research results have shown that the probability of obtaining promising results for nonstationarity in time series is spurious. This issue is discussed in greater detail by PHILLIPS (1986).

Nonstationarity observed in economic processes is not homogenous. Economic time series may be marked by nonstationarity in average or variance. A deterministic trend is observed in nonstationary processes in average, while nonstationary processes in variance are marked by a stochastic trend.
A nonstationary process in average which is characterised by stationary deviations from average P_t may be given by the following formula:

$$X_t = P_t + X_t^*, \tag{1.2}$$

$$X_t^* = \alpha X_{t-1}^* + \varepsilon_{1,t},\ |\alpha| < 1,\ \varepsilon_{1,t} \sim N(0, \sigma_1). \tag{1.3}$$

The variance and autocovariance of process X_t^* are constant. As $t \to \infty$, variance X_t approximates the constant. Process X_t^* has the ability of returning to the average level, i.e. the period after which X_t^* intersects $E(X_t^*)$ is constant (PIŁATOWSKA 2003).

If time series nonstationarity in average is eliminated, trend P_t should be subtracted from the value of the economic process, i.e. operation $X_t - P_t = \eta_t$ should be performed, where P_t is a trend component isolated with the use of a given operator.

Process Y_t which is nonstationary in variance, i.e. a process with a stochastic trend (integrated process), given by the formula:

$$Y_t = Y_{t-1} + \varepsilon_{2,t}, \; \varepsilon_{2,t} \sim N(0, \sigma_2) \tag{1.4}$$

is a stationary process in average or an integrated ARIMA(0,1,0) process.

The average of process Y_t is constant and equals zero, and variance $\mathrm{var}(Y_t)$ and autocovariance γ_i of process Y_t are a function of time *t*. As $t \to \infty$, the variance of process Y_t increases, i.e. process Y_t has a trend in variance. Process Y_t does not have the ability of returning to the average level, i.e. the period after which Y_t intersects $E(Y_t)$ is infinite because Y_t increases systematically as $t \to \infty$ and is unable to return to average $E(Y_t)$.

In a time series with a stochastic trend, successive increments have to be calculated to attain time series stationarity. In this context, the concept of integrated series should be adopted for greater convenience (CHAREMZA DEADMAN 1997).

2. Integration processes

ENGLE and GRANGER (1987) refer to a nonstationary series which may be reduced to stationarity by calculating successive increments (*d* times) as integrated series of order *d*. Integrated series x_t of order *d* are usually marked with the symbol $x_t \sim I(d)$. Therefore, if a process expressed by formula (1.4) is analysed (replacing $Y_t = y_t$), and a variable is defined with an eliminated trend:

$$\Delta y_t = y_t - y_{t-1} = \varepsilon_t, \tag{2.1}$$

then Δy_t is stationary $(y_t \sim I(1))$. If the operation of calculating second differences $\Delta(y_t - y_{t-1})$ is performed for $y_t \sim I(2)$, the resulting variables are second increments $\Delta\Delta y_t$ which are stationary.

Unit root tests are the most popular method for investigating the degree of variable integration on the assumption that it can be transformed into a stationary variable through the calculation of increments (DICKEY FULLER 1979; CHAREMZA DEADMAN 1997; PIŁATOWSKA 2003). Unit root tests are usually performed by testing the existence of autoregressive unit roots where the null hypothesis states that the process has a unit root (is integrated) $H_0 :\sim I(1)$, as opposed to an alternative hypothesis postulating that the process is stationary $H_1 :\sim I(0)$. The most popular tests in this group are (PIŁATOWSKA 2003): Dickey-Fuller (DF) test, Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test, modified Phillips-Perron (PPP) test, and Saragan-Bhargacy (SB) test with instrumental variables.

3. Nonstationarity type in time series of property prices

Evidence of autocorrelation and nonnormality in real estate data has been found (MYER WEBB 1994). Others find evidence of nonstationarity in real estate data (CHAUDHRY MYER WEBB 1999). CHAUDHRY, MYER and WEBB (1999) suggest that nonstationarity in the time series for aggregate, office, retail, and industrial is dominant when both drift and trend are included in the model. This could be logically expected, since many of the properties that are used to construct all of the real estate series have a third-party (not owner or manager) appraisal on an annual basis. All the commercial real estate time series under investigation (U.S., Canadian, U.K.) indicate clear evidence of nonstationarity. Furthermore, the presence of both a drift and trend component is indicated. The implication is that the longrun forecasts of U.S., Canadian and U.K. real estate indices (except industrial properties in nominal terms) do not diverge significantly in the long run (CHAUDHRY MYER WEBB 1999).

YOUNG and GRAFF (1995) suggest that investment risk models with infinite variance provide a better description of distributions of individual property returns. Consequently, standard statistical procedures, especially in the presence of nonstationarity, may give misleading inferences. However, advances in cointegration methodologies provide an alternative framework for investigating equilibrium price adjustments in financial and real estate time series, especially their long-run relationships. WEBB, MILES and GUILKEY (1992) suggest that appraisal-based estimates may be unbiased in the long run. Cointegration methods can be applied to appraisal-based data to study long-tenn relationships among real estate assets while accounting for idiosyncrasies in the data. The resulting estimates should provide a rich source of information to investors, particularly institutions, given the increasing involvement of institutions in real estate markets.
Research by LIANG and WEBB (1996) has identified problems with the analysis of real estate as an asset class based on return analysis. The lack of data is not just the result of a limited number of observations over time, but is also constrained by the differences in the attributes and characteristics of the data that is available from an array of sources. Each available data series reflects varying time frames and frequencies of measurement. Differences occur from variances in methodology and implied assumptions, a lack of uniformity in the technical composition of each series, the variation in the composition of property types considered, and inconsistency in the details of the transactions used or appraisal processes employed (Grissom and De Lisle 1998).

GRISSOM and DE LISLe (1997) argue that institutional ownership in real estate is relatively long term and that institutional ownership groups require analysis of more than an individual property. Thus, these methods seem especially suitable.

4. Identification of nonstationarity type in time series of property prices

4.1. Data

The descriptive characteristics of the gathered observation sets are presented in Table 1. They comprise the time series of land property prices quoted on three real estate markets in Poland. The names of variables (property prices) which will be investigated in this paper are indicated in column no. 2 of Table 1. The time series of prices are presented in Figure 1. The time series of price increments are shown in Figure 2.

Table 1

Descriptive characteristics of particular research areas

Research Object*	Variable	N. of obs.	Mean [zł]	Median [zł]	Min [zł]	Max [zł]	Stand. Dev.	Skew-ness	Kurtosis
1	2	3	4	5	6	7	8	9	10
Lok_Byd	C_lok_Byd	3584	692.76	626.03	116.18	1536.29	273.87	0.913	0.273
Lok_Che	C_lok_Che	361	959.35	1088.27	155.81	1586.85	370.25	-0.480	-1.181
Lok_Ol	C_lok_Ol	1954	1288.69	1303.00	412.00	2333.00	284.60	-0.099	0.136

* Data source: WIŚNIEWSKI (2007).

Source: own calculations.

The nonstationarity identification process began with the development of charts which present the average prices and standard deviations over time for the studied objects: *Lok_Byd*, *Lok_Che*, *Lok_Ol* (Fig. 3). It can be initially presumed that the time series is nonstationary because both the average price and standard deviation change over time (refer to *Lok_Byd*, *Lok_Ol* in Fig. 1). Figure 1 for object *Lok_Che* differs from the other, because it shows a price time series which is not marked by a distinct trend. An analysis of data in Figure 1, Figure 2 and Figure 3 is indicative of an unidentified trend and fluctuations of varying frequency. The nonstationarity type in the time series of property prices is difficult to identify in view of the above data. For this reason, the following methods were employed to identify the nonstationarity type of the time series of property prices:

1) comparison of the autocorrelation function (ACF) and the partial autocorrelation function (PACF),
2) specification of Ljung-Box *Q-statistics* values,
3) unit root tests.

ACF, PACF and Ljung-Box Q-statistics tests were performed for all investigated objects. The results reported for object *Lok_Byd* are presented in Table 2. Differences in functions ACF and PACF for objects: *Lok_Byd, Lok_Che* and *Lok_Poz* are shown in Figure 4.

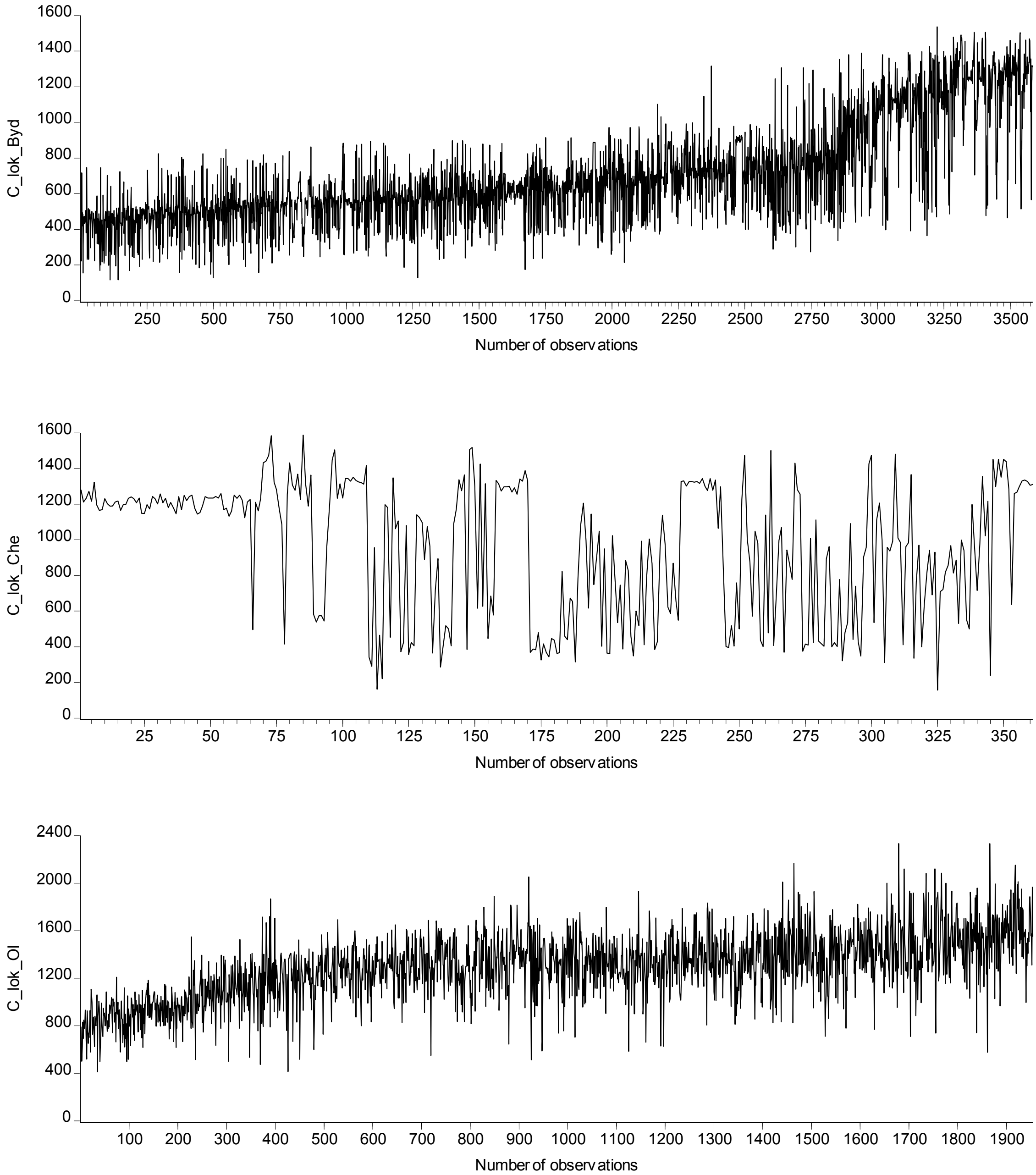

Fig. 1. Price time series for objects: *Lok_Byd, Lok_Che, Lok_Ol. Source: o*wn calculations.

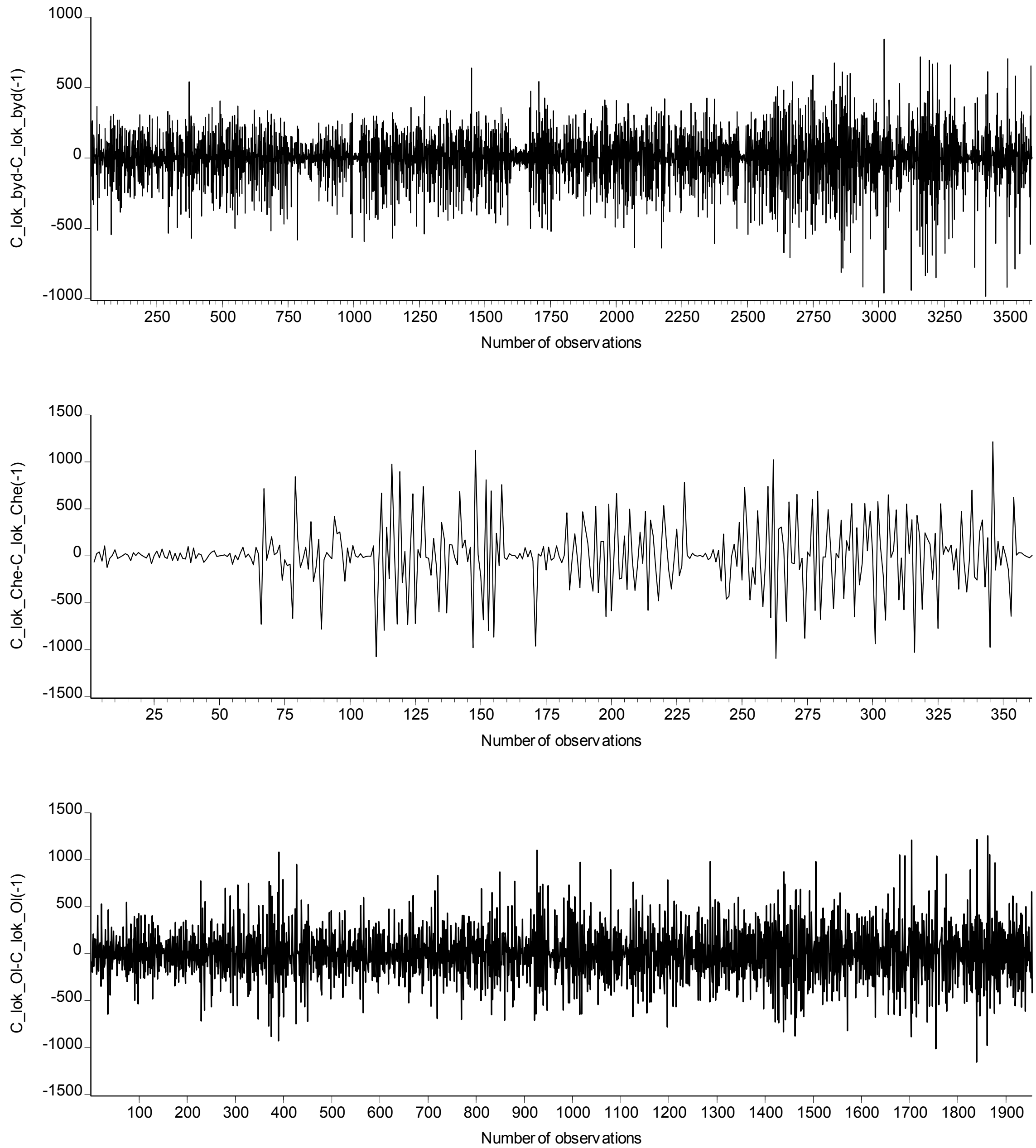

Fig. 2. Time series of price increments for objects: *Lok_Byd*, *Lok_Che*, *Lok_Ol*. *Source: own calculations.*

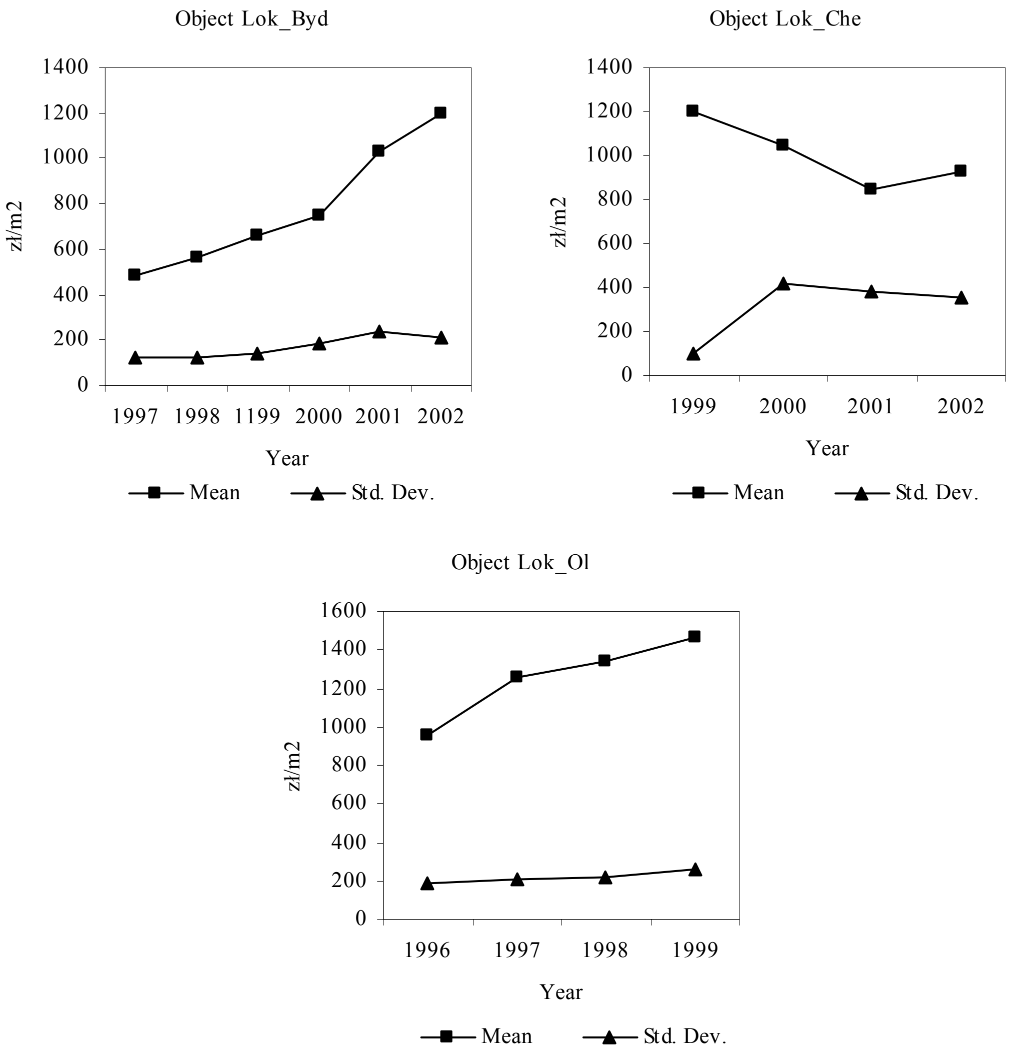

Fig. 3. Average price and standard deviation in each year for objects: *Lok_Byd, Lok_Che, Lok_Ol. Source: own calculations.*

Table 2

Autocorrelation function (ACF), partial autocorrelation function (PACF) and Q_{LB} values of Ljunga-Box statistics of the time series of prices $\left(C_t\right)$, first differences $\left(\Delta C_t\right)$ and residuals from linear trend $\left(C_t - T_L\right)$ for object *Lok_Byd Source:* own calculations

k	C_t				ΔC_t				C_t-T_L			
	ACF	*PACF*	Q_{LB}	*p*	*ACF*	*PACF*	Q_{LB}	*p*	*ACF*	*PACF*	Q_{LB}	*p*
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
1	0.77	0.77	2127	0.000	-0.4	-0.4	574.13	0.000	0.383	0.383	527.18	0.000
2	0.723	0.32	4003.6	0.000	-0.061	-0.263	587.39	0.000	0.259	0.131	767.93	0.000
3	0.704	0.217	5784.9	0.000	-0.03	-0.212	590.52	0.000	0.21	0.088	926.42	0.000
4	0.7	0.182	7543.2	0.000	0.023	-0.135	592.41	0.000	0.198	0.084	1067	0.000
5	0.684	0.112	9225	0.000	-0.014	-0.116	593.14	0.000	0.157	0.033	1155.9	0.000
6	0.677	0.102	10872	0.000	-0.019	-0.119	594.46	0.000	0.135	0.03	1221.9	0.000
7	0.678	0.108	12523	0.000	-0.001	-0.109	594.47	0.000	0.137	0.046	1289.6	0.000
8	0.678	0.097	14177	0.000	0.013	-0.083	595.07	0.000	0.14	0.046	1360	0.000
9	0.673	0.072	15804	0.000	-0.004	-0.075	595.12	0.000	0.127	0.028	1417.9	0.000
10	0.669	0.063	17412	0.000	-0.004	-0.071	595.18	0.000	0.118	0.025	1468.3	0.000

Source: own calculations.

Fig. 4 indicates that the value of the autocorrelation function (ACF) of prices $\left(C_t\right)$ for a single lag (object *Lok_Byd*) equals 0.77, and for objects *Lok_Che* and *Lok_Poz* – 0.51 and 0.45, respectively. For subsequent lags, it decays exponentially in object *Lok_Che*, while a very slow decay was reported for the remaining objects. The calculation of the first differences $\left(\Delta C_t\right)$ results in a first-order negative autocorrelation reaching -0.38 (cf. ACF for object *Lok_Byd* in Fig. 4) and –0.44 and – 0.50, respectively (cf. ACF for objects *Lok_Che* and *Lok_Ol* in Fig. 4); this value decayed rapidly for successive lags. In the partial autocorrelation function (PACF) (cf. PACF in Fig. 4), this process is expressed by a clearly pronounced negative autocorrelation for the several first lags. The above indicates that a real data-generating model is a model with a linear trend and a weak first-order autoregression effect, implying that we are dealing with nonstationarity in average.

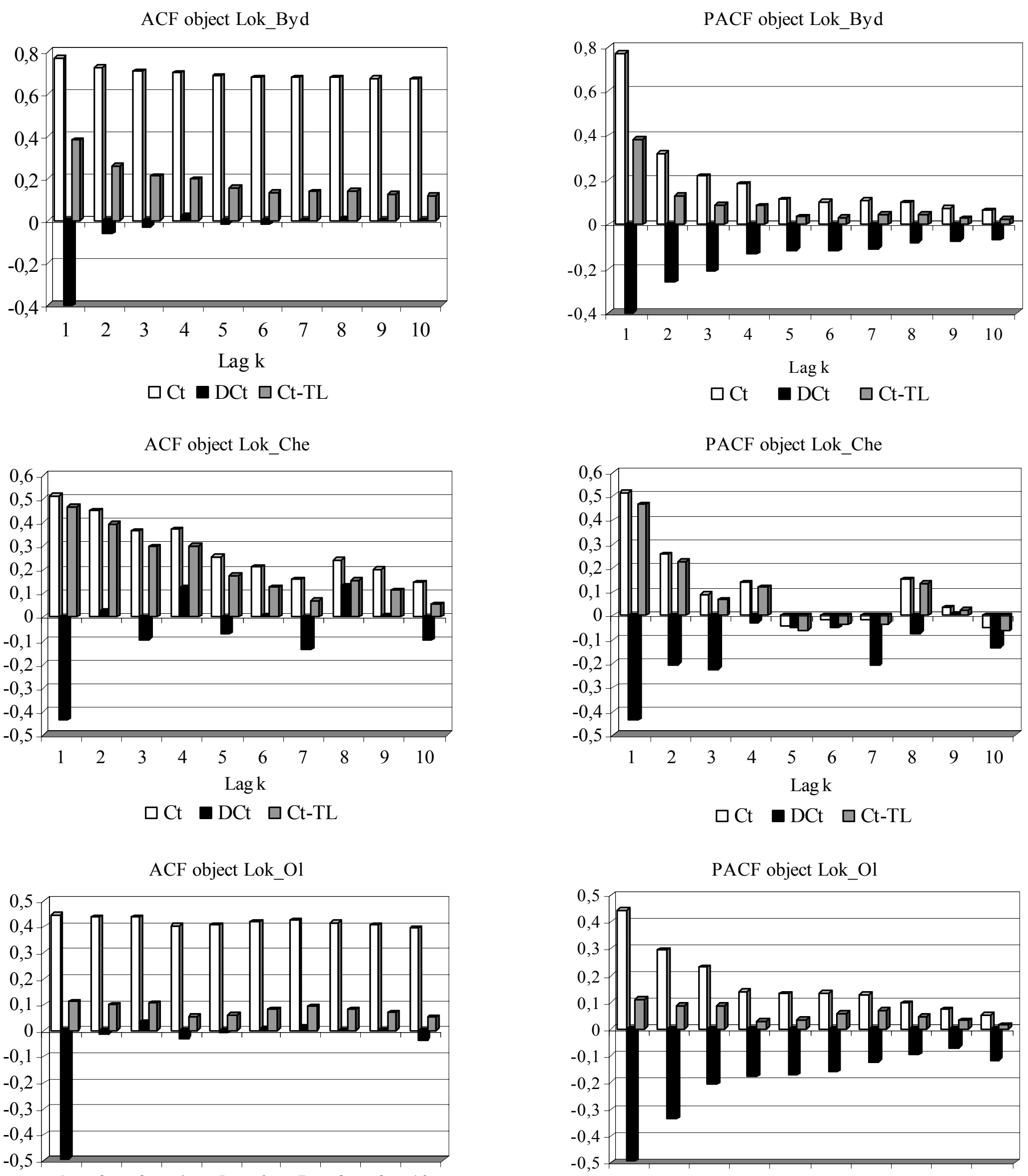

Fig. 4. Autocorrelation function (ACF), partial autocorrelation function (PACF) of the time series of prices $\left(C_t\right)$, first differences $\left(\Delta C_t\right)$ and residuals from linear trend $\left(C_t - T_L\right)$ for objects: *Lok_Byd, Lok_Che, Lok_Ol*. *Source:* own calculations.

Nonstationarity should be eliminated if time variable *t* is introduced to the model. As regards objects *Lok_Byd* and *Lok_Ol*, the introduction of variable *t* significantly reduces the autocorrelation effect in the price series (cf. $C_t - T_L$ in Fig. 4), but does not eliminate it entirely. This effect is observed in object *Lok_Che* (cf. ACF and PACF in Fig. 4). It is practically not reported for lag k = 10. The above validates the assumption that a price series is nonstationary in average with an autoregression effect.

4.2. Integration process testing – unit root tests

Nonstationary processes in average (processes with a deterministic trend) and nonstationary processes in variance (processes with a stochastic trend) have different characteristics and suggest different dynamics of economic variables. Therefore, the given process type has to be identified. Below is an overview of an analysis to determine the type of nonstationarity based on unit root tests.

Autoregressive unit root tests propose parameter ρ in the following model (BHARGAVA 1986; PIŁATOWSKA 2003):

$$C_t = d_t + u_t \tag{4.1}$$

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{4.2}$$

where: d_t is a deterministic component, ε_t is white noise. $d_t = 0$ implies the absence of a deterministic component, $d_t = \alpha$ implies the presence of a constant, $d_t = \alpha + \beta t$ implies the presence of a linear trend.

Assuming that $H_0 : \rho = 1$ i $d_t = 0$, process C_t is a random walk without drift. If $|\rho| < 0$, then the process is stationary around an average (cf. BHARGAVA 1986). If $d_t = \alpha$, where $H_0 : \rho = 1$, process C_t is a random walk without drift, when $|\rho| < 0$, the process is stationary around a non-zero average. If $d_t = \alpha + \beta t$, on the assumption that $H_0 : \rho = 1$, process C_t is a random walk with drift, i.e. $\Delta C_t = \beta + \varepsilon_t$, and if $|\rho| < 0$, process C_t has a linear trend with coefficient $\beta(1-\rho)$ (cf. PIŁATOWSKA 2003).

Unit root tests verify the hypothesis on the type of nonstationarity in time series. If the null hypothesis $H_0 : \rho = 1$ is rejected, the process belongs to a class of process with stationary deviations from the deterministic trend, and a model of such a process should be built for price levels. If the null hypothesis is not rejected, we are dealing with a process with stationary increments whose model should be built for respective increments.

The Augmented Dickey-Fuller test (ADF) is one of the basic unit root tests. Dickey-Fuller t_{DF} statistics is a quotient of estimate $\left(\hat{\rho}-1\right)$ and standard error of estimate $s\left(\hat{\rho}\right)$ (PIŁATOWSKA 2003):

$$t_{DF}=\frac{\left(\hat{\rho}-1\right)}{s\left(\hat{\rho}\right)} \tag{4.3}$$

in the following model:

$$C_t=\alpha+\left(\hat{\rho}-1\right)C_{t-1}+\sum_{i=1}^{p-1}\hat{\rho}\Delta C_{t-i}+\varepsilon_t \tag{4.4}$$

where: C_t is a process in the form of $C_t=\alpha+\sum_{i=1}^{p}\rho_i C_{t-i}+\varepsilon_t$, and p is the autoregression order.

PHILLIPS and PERRON (1988) modified the Dickey-Fuller test by accounting for a residual autocorrelation and adjusting standard DF statistics. To account for the residual autocorrelation effect, they proposed a procedure of modifying standard statistics after the estimation of model parameters, without the need to estimate additional parameters. The PHILLIPS-PERRON (PP) test (1988) is based on the below statistics:

$$t_{PP}=t_{DF}\left(\frac{s_0}{f_0}\right)^{1/2}-\frac{T\left(f_0-s_0\right)\left(s\left(\hat{\rho}\right)\right)}{2f_0^{1/2}s} \tag{4.5}$$

where: s_0 is a consistent estimator of residual variance (calculated as $\left(T-k\right)s^2/T$, where k is the number of regressors (lags)), f_0 is the estimator of the spectral density function at zero frequency, T is the number of observations, $s\left(\hat{\rho}\right)$ is the standard error of estimate, and s is the standard error of a regression test.

PIŁATOWSKA (2003) also notes that an adequate number of lags in the ADF test should be selected as it has been observed that the power and the number of rejections of the true null hypothesis $H_0:\rho=1$ is determined by the number of first increment lags (cf. AGIAKOGLOU, NEWBOLD 1992). NG and PERRON (1995) postulated that the number of lags in the ADF test should be determined by: identifying the constant number of lags k in an arbitrary manner subject to the number of observations, identifying the constant number of lags k as a function of the number of observations, e.g. $k=12\left(T/100\right)^{1/4}$ (SCHWERT 1989) or $k=T^{1/4}$ (DIEBOLD, NERLOVE 1990), relying on Akaike information criteria (AIC) and Schwarz Bayesian information criteria (BIC), and selecting k in line with the top-down approach or the bottom-up approach.

Table 3

Results of verification of the hypothesis $H_0 : C_t \sim I(1)$ with the first unit root for object *Byd_lok*. Augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP)

k	C_lok_Byd			ln(C_lok_Byd)		
	Model with constant	**Model with constant and trend**	**Model without constant and trend**	**Model with constant**	**Model with constant and trend**	**Model without constant and trend**
1	**2**	**3**	**4**	**5**	**6**	**7**
			ADF test			
0	-5.24***	-10.96***	**-1.38**	-6.01***	-14.30***	**0.02**
1	-31.06***	-17.64***	-37.57***	-30.60***	-9.87***	-41.67***
2	-22.20***	-13.34***	-25.71***	-23.78***	-8.52***	-29.76***
3	-17.49***	-10.86***	-19.85***	-18.95***	-7.18***	-22.82***
4	-13.01***	-7.94***	-14.70***	-14.67***	-5.43***	-17.36***
5	-11.12***	-7.00***	-12.42***	-12.65***	-5.09***	-14.67***
6	-9.71***	-6.42***	-10.71***	-10.26***	-4.28***	-11.76***
7	-7.45***	-5.00***	-8.18***	-8.76***	-4.20***	-9.85***
8	-4.48***	**-2.87**	-4.94***	-6.62***	-3.62**	-7.30***
$d_t = \alpha$	**5.06**	**9.01**	**-**	**6.02**	**14.30**	**-**
$d_t = \alpha + \beta t$	-	**9.60**	**-**	**-**	**12.91**	**-**
$adjR^2$	0.30	0.33	0.30	0.34	0.37	0.33
			PP test			
0	-21.48***	-35.77***	-7.41***	-26.59***	-44.25***	**-1.40**
$d_t = \alpha$	**20.01**	**24.38**	**-**	**26.54**	**44.08**	**-**
$d_t = \alpha + \beta t$	-	**26.93**	**-**	**-**	**32.33**	**-**
$adjR^2$	0.11	0.26	0.02	0.16	0.35	0.00
			Critical test values for a given significance level			
1%	-3.43	-3.96	-2.57	-3.43	-3.96	-2.57
5%	-2.86	-3.41	-1.94	-2.86	-3.41	-1.94
10%	-2.57	-3.13	-1.62	-2.57	-3.13	-1.62

***, **, * - rejection of H_0 at a significance level of 1%, 5% and 10%, respectively. Cases when H_0 was not rejected are written in bold, $adjR^2$ - corrected model fitting coefficient. *Source:* own calculations.

Table 3 presents the results of first unit root verification with the use of ADF and PP tests in object *Lok_Byd*, with different assumptions concerning price series transformations (finding the logarithm – columns 5, 6 and 7), the presence of a constant in the model (columns 2 and 5), the presence of a constant and a trend in the model (columns 3 and 6) and a model without a constant or a trend (columns 4 and 7). Table 4 presents the results of second unit root verification for the same set of assumptions which apply to Table 3. The number of lags in the ADF test was selected based on dependency $k = T^{1/4}$ (DIEBOLD, NERLOVE 1990). Calculations for the remaining objects are presented in Annexes 1a and 1c.

The results presented in Table 3 point to a lack of uniqueness as regards stationarity in average. The ADF test performed on original (non-logarithmed) data for the model with a constant and the model without a trend or a constant indicates that unit root hypothesis H_0 is rejected in most cases, thus assuming the occurrence of nonstationarity in average. The above is validated by the PP test. An analysis of logarithmed property price series leads to similar conclusions. As regards the model with a constant and a linear trend, nonstationarity in variance to the second lag is rejected by the ADF test. Test values for the constant in the model with a constant (columns 2 and 5), and for the constant and the trend (columns 3 and 6) are significant. The above is validated by the PP test. As regards all objects, the hypothesis ($H_0 : C_t \sim I(1)$) with the first unit root may be adopted in models without a constant or a trend (columns 4 and 7 in Table 3, columns 4 and 7 in Annex 1a and 1c) for lag = 0. In this case, the characteristic features of the model without a constant or a trend indicate that the time series is a random walk without drift.

The indicated lack of uniqueness prompted the need to calculate the second unit root. Calculation results are presented in Table 4. Calculations for the remaining objects are presented in Annexes 1b and 1d.

Tests to determine the degree of integration, i.e. calculation of the second unit root, brought unexpected results. The values presented in Table 4 for both tests are positive and high in most cases. It should be noted that the value of $adjR^2$ statistics in Table 4 is much higher (nearly two-fold) than in Table 3. Test values indicated in Table 4 point to overdifferencing, i.e. the use of the subtraction operator too many times (cf. CHAREMZA, DEADMAN 1997).

The KPSS stationarity test (KWIATKOWSKI et al. 1992) was performed to validate the process of overdifferencing.

Table 4

Results of verification of the hypothesis $H_0 : C_t \sim I(2)$ with the second unit root for object *Lok_Byd*. Augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP) [1].

k	ΔC_lok_Byd			ln(ΔC_lok_Byd)		
	Model with constant	**Model with constant and trend**	**Model without constant and trend**	**Model with constant**	**Model with constant and trend**	**Model without constant and trend**
1	**2**	**3**	**4**	**5**	**6**	**7**
			ADF test			
0	-31.41***	-31.41***	-31.42***	-32.95***	-32.94***	-32.95***
1	**19.36**	**19.36**	**19.36**	**21.48**	**21.48**	**21.48**
2	**16.40**	**16.40**	**16.40**	**18.33**	**18.32**	**18.33**
3	**13.97**	**13.97**	**13.97**	**15.75**	**15.74**	**15.75**
4	**12.16**	**12.16**	**12.16**	**13.74**	**13.74**	**13.74**
5	**10.41**	**10.41**	**10.41**	**11.79**	**11.79**	**11.79**
6	**8.47**	**8.47**	**8.47**	**10.00**	**10.00**	**10.00**
7	**6.43**	**6.43**	**6.43**	**7.81**	**7.81**	**7.81**
8	**4.48**	**4.48**	**4.48**	**4.76**	**4.76**	**4.76**
$d_t = \alpha$	**0.35**	**0.05**	**-**	**0.34**	**0.17**	**-**
$d_t = \alpha + \beta t$	-	**0.14**	**-**	**-**	**0.00**	**-**
$adjR^2$	0.75	0.75	0.75	0.76	0.76	0.76
			PP test			
0	-91.42***	-91.41***	-91.43***	-92.19***	-92.17***	-92.20***
$d_t = \alpha$	**0.15**	**0.03**	**-**	**0.13**	**0.09**	**-**
$d_t = \alpha + \beta t$	-	**0.04**	**-**	**-**	**-0.03**	**-**
$adjR^2$	0.70	0.70	0.70	0.70	0.70	0.70

[1] – critical test value as in Table 3.

***, **, * - rejection of H_0 at a significance level of 1%, 5% and 10%, respectively.

Cases when H_0 was not rejected are written in bold, $adjR^2$ - corrected model fitting coefficient. *Source:* own calculations.

Test results (Tables 5 and 6) validate the process of overdifferencing. Table 5 presents calculations indicating that the null hypothesis which postulates that process C_t is stationary should be rejected. This implies that the price time series is nonstationary which, following the rejection of the first unit root hypothesis (Table 3), points to nonstationarity in average. An analysis of data in Table 6 (KPSS test to determine increments in land property prices) shows that the time series of first price increments is stationarity. Yet statistical values are nearly equal to zero. The above clearly indicates that the use of the subtraction operator leads to overdifferencing.

Table 5

Results of verification of the hypothesis $H_0 : C_t$ is stationary for object *Lok_Byd, Lok_Che i Lok_Ol*. KPSS test

	Model with constant	**Model with constant and trend**	**Model with constant**	**Model with constant and trend**
1	***2***	***3***	***5***	***6***
	C_gr_Byd		ln(C_lok_Byd)	
	LM-Statistic*			
	30.04	**4.63**	31.97	2.48
	t-Statistic**			
$d_t = \alpha$	151.43 *(0.00)*	55.75 *(0.00)*	989.24 *(0.00)*	669.02 *(0.00)*
$d_t = \alpha + \beta t$	-	68.27 *(0.00)*	-	64.04 *(0.00)*
	C_lok_Che		ln(C_lok_Che)	
	LM-Statistic*			
	1.39	**0.36**	1.30	0.38
	t-Statistic**			
$d_t = \alpha$	49.23 *(0.00)*	30.34 *(0.00)*	261.99 *(0.00)*	139.72 *(0.00)*
$d_t = \alpha + \beta t$	-	-5.44 *(0.00)*	-	-4.93 *(0.00)*
	C_lok_Ol		ln(C_lok_Ol)	
	LM-Statistic*			
	16.62	**1.78**	15.87	2.25
	t-Statistic**			
$d_t = \alpha$	200.16 *(0.00)*	96.27 *(0.00)*	1310.92 *(0.00)*	779.83 *(0.00)*
$d_t = \alpha + \beta t$	-	32.82 *(0.00)*	-	31.76 *(0.00)*
	Wartości krytyczne testu LM dla danego poziomu istotności*			
1%	0.74	0.22	0.74	0.22
5%	0.46	0.15	0.46	0.15
10%	0.35	0.12	0.35	0.12

* - Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

** p-value for Student's t-statistics at a significance level of 0.05 is given in brackets. Cases when H_0 was rejected are indicated in bold. *Source:* own calculations

Table 6

Results of verification of the hypothesis $H_0 : \Delta C_t$ is stationary for object *Lok_Byd, Lok_Che i Lok_Ol*. KPSS test

	Model with constant	**Model with constant and trend**	**Model with constant**	**Model with constant and trend**
1	***2***	***3***	***5***	***6***
	ΔC_lo_Byd		ln(ΔC_lok_Byd)	
		LM-Statistic*		
	0.00**	0.00**	0.00**	0.00**
		t-Statistic***		
$d_t = \alpha$	0.09 *(0.93)*	0.005 *(0.995)*	0.07 *(0.94)*	0.03 *(0.98)*
$d_t = \alpha + \beta t$	-	-0.04 *(0.96)*	-	0.01 *(0.99)*
	ΔC_lok_Che		ln(ΔC_lok_Che)	
		LM-Statistic*		
	0.02**	0.01**	0.01**	0.01**
		t-Statistic***		
$d_t = \alpha$	0.004 *(0.996)*	-0.14 *(0.88)*	0.002 *(0.998)*	-0.12 *(0.90)*
$d_t = \alpha + \beta t$	-	0.17 *(0.87)*	-	0.14 *(0.89)*
	ΔC_lok_Ol		ln(ΔC_lok_Ol)	
		LM-Statistic*		
	0.00**	0.00**	0.00**	0.00**
		t-Statistic***		
$d_t = \alpha$	0.06 *(0.95)*	0.06 *(0.96)*	0.06 *(0.95)*	0.06 *(0.95)*
$d_t = \alpha + \beta t$	-	-0.03 *(0.97)*	-	-0.04 *(0.97)*

* - Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1). Critical test value as in Table 5.

** - Cases when H_0 was accepted are marked.

*** p-value for Student's t-statistics at a significance level of 0.05 is given in brackets. *Source:* own calculations

5. Conclusions

In recent years, the residential property market in Poland has undergone dynamic change which resulted in a steep increase in property prices. These changes have specific implications for the time series of residential property prices.

The Polish property market is undergoing structural and functional transformations. According to the conducted study, the observed nonstationarity in price time series is directly proportional to the intensity of changes noted on the residential property market. This dependency is not only indicative of changes in the probability distribution function over time due to changes in the average price level. This dependency, which validates nonstationarity in the time series of residential property prices, is determined by structural changes in the property market system.

The results of this study indicate that the analysed time series of land property prices are roughly nonstationary in average, and increment calculation results in overfitting. ADF, PP and KPSS tests have shown that the time series of land property prices is not integrated, implying that it may not be transformed into a stationary time series through increment calculation. It should, however, be noted that the inability to determine integration in the time series of property prices could be due to the low power of the applied tests (cf. PHILLIPS, PERRON 1988; DIEBOLD, NERLOVE 1990; CHAREMZA, DEADMAN 1997; PIŁATOWSKA 1997).

The identified nonstationarity in average should be eliminated by subtracting trend P_t, i.e. by performing operation $X_t - P_t = \eta_t$, where P_t is a trend component isolated with the use of a given operator. The isolation of component P_t is also significant for another reason – the isolated component can be introduced to the model. If components P_t are isolated for various processes, the dependency between those components, i.e. components describing changes in average values, can be investigated. Yet to date, a universal and fully objective method for selecting the trend form has not been developed. The following methods can be proposed: the growth curve method, multinomial function of time variable, investigation of absolute and relative increments, trigonometric polynomial, various filters (moving average filters, recursive filters, exponential smoothing filters). In general, the simplest functions are recommended, provided that they are effective.

An analysis of the internal structure of time series of property prices is one of the main stages of an analysis of the gathered data. It aims to determine the type of relationships found in the analysed data. As regards classical statistical models of property prices (e.g. regressive) based on nonstationary series, the type of nonstationarity has to be correctly identified at the first stage for the appropriate parameter estimation procedures to be applied at the successive stage. A diagnosis of time series of property prices which are used to, for example, forecast property value in the context of a given trend, and an analysis of variable integration processes support the forecasting process. The above particularly applies to situations in which the forecasting strategy is based on the process of simulating the property market model, on the assumption that variables will take up the values of a given probability distribution function. The use of various prognostic methods based on stationary series is always easier, and the obtained results are marked by lower uncertainty due to the manner in which data are processed by a prognostic model.

The application of classical methods for forecasting property value under such circumstances produces apparent regressions which could adversely affect the quality of the forecast. Statistical analyses for forecasting property value should be conducted with the use of tools that account for nonstationarity in the course of the estimation process.

The identification of nonstationarity type is of paramount importance for the selection of the right method of forecasting property value. For the needs of the forecasting process, short- and long-term relationships have to be determined. Those analyses are significant because, on the one hand, they identify autocorrelation dependencies in data and validate property value forecasts on the side of data-generating processes, while on the other hand, the identification of the type of the dependency, and the power and stability of its effect, supports the correct selection of prognostic process parameters (e.g. the prognostic horizon).

The lack of uniqueness in the process of identifying the nonstationarity type in the time series of residential property prices indicates that an analysis of the time series of residential property prices in Poland may be a source of information which supports the identification of dependences between data, and as such, it should condition the choice of the appropriate research methods.

The resulting estimates should provide a rich source of information to owners, investors, particularly institutions, given the increasing involvement of institutions in real estate markets.

Annex 1a. Results of verification of the hypothesis $H_0 : C_t \sim I(1)$ with the first unit root for object *Lok_Che*. Augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP).

k	**C_lok_Che**			**ln(C_lok_Che)**		
	Model with constant	**Model with constant and trend**	**Model without constant and trend**	**Model with constant**	**Model with constant and trend**	**Model without constant and trend**
1	**2**	**3**	**4**	**5**	**6**	**7**
			ADF test			
0	-5.03***	-5.30***	**-1.33**	-5.14***	-5.38***	**-0.24**
1	-5.15***	-4.47***	-10.24***	-5.38***	-4.71***	-12.05***
2	**-2.38**	**-1.94**	-5.23***	-2.80**	**-2.37**	-6.32***
3	**-1.77**	**-1.43**	-3.78***	**-1.98**	**-1.65**	-4.33***
4	**0.79**	**1.01**	-0.42	**0.46**	**0.66**	**-0.93**
$d_t = \alpha$	**4.84**	**4.68**	**-**	**5.13**	**5.33**	**-**
$d_t = \alpha + \beta t$	-	**-1.68**	**-**	**-**	**-1.62**	**-**
$adjR^2$	0.30	0.31	0.26	0.34	0.35	0.30
			PP test			
0	-10.70***	-11.21***	-3.41***	-11.89***	-12.40***	**-0.73**
$d_t = \alpha$	**9.98**	**9.43**	**-**	**11.86**	**12.26**	**-**
$d_t = \alpha + \beta t$	-	**-2.98**	**-**	**-**	**-3.03**	**-**
$adjR^2$	0.24	0.26	0.03	0.28	0.30	0.00
			Critical test values for a given significance level			
1%	-3.47	-4.01	-2.58	-3.47	-4.01	-2.58
5%	-2.88	-3.43	-1.94	-2.88	-3.43	-1.94
10%	-2.57	-3.14	-1.62	-2.57	-3.14	-1.62

***, **, * - rejection of H_0 at a significance level of 1%, 5% and 10%, respectively.

Cases when H_0 was not rejected are written in bold, $adjR^2$ - corrected model fitting coefficient. *Source:* own calculations

Annex 1b. Results of verification of the hypothesis $H_0 : C_t \sim I(2)$ with the second unit root for object *Lok_Che*. Augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP) [(1)].

	ΔC_lok_Che			**ln(ΔC_lok_Che)**		
k	**Model with constant**	**Model with constant and trend**	**Model without constant and trend**	**Model with constant**	**Model with constant and trend**	**Model without constant and trend**
1	**2**	**3**	**4**	**5**	**6**	**7**
			ADF test			
0	-11.19***	-11.19***	-11.20***	-11.25***	-11.24***	-11.27***
1	**3.96**	**3.97**	**3.96**	**4.12**	**4.13**	**4.13**
2	**2.58**	**2.60**	**2.59**	**2.56**	**2.57**	**2.56**
3	**1.11**	**1.12**	**1.11**	**0.98**	**0.99**	**0.98**
4	**0.99**	**1.00**	**0.99**	**0.58**	**0.59**	**0.58**
$d_t = \alpha$	**0.02**	**-0.39**	**-**	**0.01**	**-0.35**	**-**
$d_t = \alpha + \beta t$	-	**0.45**	**-**	**-**	**0.41**	**-**
$adjR^2$	0.74	0.74	0.74	0.76	0.76	0.76
			PP test			
0	-30.17***	-30.13***	-30.21***	-31.31***	-31.27***	-31.35***
$d_t = \alpha$	**0.02**	**-0.21**	**-**	**0.01**	**-0.19**	**-**
$d_t = \alpha + \beta t$	-	**0.25**	**-**	**-**	**0.22**	**-**
$adjR^2$	0.72	0.72	0.72	0.73	0.73	0.73

(2) – critical test value as in annex 1a.

***, **, * - rejection of H_0 at a significance level of 1%, 5% and 10%, respectively.

Cases when H_0 was not rejected are written in bold, $adjR^2$ - corrected model fitting coefficient. *Source:* own calculations

Annex 1c. Results of verification of the hypothesis $H_0 : C_t \sim I(1)$ with the first unit root for object *Lok_Ol*. Augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP).

k	C_lok_Ol			ln(C_lok_Ol)		
	Model with constant	**Model with constant and trend**	**Model without constant and trend**	**Model with constant**	**Model with constant and trend**	**Model without constant and trend**
1	**2**	**3**	**4**	**5**	**6**	**7**
			ADF test			
0	-6.03***	-11.09***	**-0.53**	-6.00***	-10.19***	**0.27**
1	-22.49***	-8.70***	-36.90***	-23.11***	-10.89***	-37.94***
2	-17.12***	-7.29***	-24.05***	-17.87***	-9.35***	-24.87***
3	-13.06***	-5.71***	-17.28***	-14.01***	-7.77***	-18.23***
4	-11.43***	-5.57***	-14.41***	-12.06***	-7.25***	-14.94***
5	-9.82***	-5.30***	-11.92***	-10.02***	-6.43***	-12.00***
6	-7.55***	-4.29***	-8.97***	-7.17***	-4.64***	-8.48***
7	-4.54***	**-2.49**	-5.39***	-4.24***	-2.65	-5.03***
$d_t = \alpha$	**6.02**	**10.91**	**-**	**6.01**	**10.20**	**-**
$d_t = \alpha + \beta t$	-	**9.23**	**-**	**-**	**8.16**	**-**
$adjR^2$	0.43	0.45	0.42	0.44	0.45	0.43
			PP test			
0	-27.43***	-38.43***	-5.00***	-27.28***	-37.40***	**-0.72**
$d_t = \alpha$	**26.80**	**34.95**	**-**	**27.26**	**37.35**	**-**
$d_t = \alpha + \beta t$	-	**22.87**	**-**	**-**	**21.78**	**-**
$adjR^2$	0.28	0.43	0.01	0.28	0.42	0.00
			Critical test values for a given significance level			
1%	-3.43	-3.96	-2.57	-3.43	-3.96	-2.57
5%	-2.86	-3.41	-1.94	-2.86	-3.41	-1.94
10%	-2.57	-3.13	-1.62	-2.57	-3.13	-1.62

***, **, * - rejection of H_0 at a significance level of 1%, 5% and 10%, respectively. Cases when H_0 was not rejected are written in bold, $adjR^2$ - corrected model fitting coefficient. *Source:* own calculations

Annex 1d. Results of verification of the hypothesis $H_0 : C_t \sim I(2)$ with the second unit root for object *Lok_Ol*. Augmented Dickey-Fuller test (ADF) and Phillips-Perron test (PP) (2).

k	ΔC_lok_Ol			ln(ΔC_lok_Ol)		
	Model with constant	**Model with constant and trend**	**Model without constant and trend**	**Model with constant**	**Model with constant and trend**	**Model without constant and trend**
1	**2**	**3**	**4**	**5**	**6**	**7**
			ADF test			
0	-26.56***	-26.55***	-26.56***	-26.45***	-26.45***	-26.45***
1	**17.29**	**17.29**	**17.29**	**17.31**	**17.31**	**17.31**
2	**14.58**	**14.57**	**14.57**	**14.53**	**14.53**	**14.53**
3	**12.55**	**12.55**	**12.55**	**12.35**	**12.35**	**12.34**
4	**10.57**	**10.57**	**10.57**	**10.22**	**10.22**	**10.22**
5	**8.47**	**8.47**	**8.46**	**8.04**	**8.04**	**8.03**
6	**6.31**	**6.31**	**6.30**	**6.02**	**6.02**	**6.01**
7	**4.24**	**4.24**	**4.24**	**4.12**	**4.12**	**4.12**
$d_t = \alpha$	**0.42**	**0.34**	**-**	**0.45**	**0.45**	**-**
$d_t = \alpha + \beta t$	-	**-0.15**	**-**	**-**	**-0.26**	**-**
$adjR^2$	0.81	0.81	0.81	0.81	0.81	0.81
			PP test			
0	-75.76***	-75.74***	-75.78***	-75.67***	-75.66***	-75.69***
$d_t = \alpha$	**0.12**	**0.09**	**-**	**0.13**	**0.12**	**-**
$d_t = \alpha + \beta t$	-	**-0.04**	**-**	**-**	**-0.07**	**-**
$adjR^2$	0.75	0.75	0.75	0.75	0.75	0.75

(2) – critical test value as in annex 1c.

***, **, * - rejection of H_0 at a significance level of 1%, 5% and 10%, respectively.

Cases when H_0 was not rejected are written in bold, $adjR^2$ - corrected model fitting coefficient. *Source:* own calculations.

3.2. THE SIGNIFICANCE OF REAL ESTATE ATTRIBUTES IN THE PROCESS OF DETERMINING LAND FUNCTION WITH THE USE OF THE ROUGH SET THEORY

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Key words: *rough set theory, value tolerance relation,, significance of attributes, land use function*

Abstract

This paper discusses the methodology for determining the effect that real estate attributes have on the selection of the appropriate land use function. An analysis of a set of real estate transactions generated a group of attributes which significantly affect the designation and, consequently, the value of property. The significance of each real estate attribute was assessed with the use of the rough set theory combined with the value tolerance relation.

Each real estate attribute is selected and its significance is assessed based on the results of a statistical analysis. A number of conditions have to be met for the statistical analysis to produce reliable results. A sufficient set of real estate data has to be collected (the larger the database, the more accurate the results), real estate attributes have to be encoded on a corresponding scale (a uniform scale for all attributes is preferred), the relations, functional dependencies and the distribution of all statistical model parameters have to be investigated.

The applied method for determining the effect that real estate attributes have on the selection of the appropriate land use function (and property valuation), based on the rough set theory, poses an alternative to statistical analyses deployed in real estate market surveys.

1. Introduction

The decision making process in real estate management is governed by the possibilities and limitations of the applied research methods which determine the optimal manner of land use and management. There are no uniform principles for determining strategic decisions, which inhibits property development and leads to material losses due to the absence of well-guided decisions or the lack of any decisions at all. The diversity and imprecision of attributes (real estate features) and a large and multidimensional set of data makes real estate planning and management a complex and time-consuming process which is marked by a high degree of risk. If decisional uncertainty concerns various elements of the decision process (e.g. occurrence of events, significance of events, usefulness of decisions, significance of decisions, criteria, etc.), then popular statistical methods may not deliver measurable results.

The method based on the rough set theory accounts for the specific features of real estate data. Developed by a Polish information technology engineer, Professor Zdzisław Pawlak, the theory is used to test imprecision, vagueness and uncertainty in the process of data analysis.

Although the rough set theory is a relatively young approach, it has a growing number of scientific applications (including DEJA 2000, KOMOROWSKI ET AL. 1999, MRÓZEK, PŁONKA 1999, POLKOWSKI AND SKOWRON 1998A, POLKOWSKI AND SKOWRON 1998B, PAWLAK 1997, NUTECH SOLUTION 2005, SŁOWIŃSKI 1992). It is used in medicine, pharmacology, economics, banking, chemistry, sociology, acoustics, linguistics, general engineering, neuroengineering and machine diagnostics. Several studies published in the recent years discuss rough set theory applications in geography and spatial planning (including D'AMATO 2004, 2006, 2007, 2008 KOTKOWSKI AND RATAJCZAK. 2002, RENIGIER 2006, RENIGIER-BIŁOZOR 2008a, 2008b, RENIGIER – BIŁOZOR and BIŁOZOR 2007, 2008).

A classical rough set theory analyses data represented in the form of qualitative attributes. The specific features of real estate attributes show a high level of variability in the method of encoding each attribute, and some key attributes are expressed on a ratio scale, such as the price or area, where the coding method should not be modified due to the risk of data loss. In view of the above, the authors have proposed to integrate the valued tolerance relation (based on the fuzzy set theory) with the classical rough set theory to fully deploy and analyze real estate market data. This approach will make the rough set theory a more flexible tool for exploring data and analyzing observations expressed in quantitative form on a ratio scale.

2. Procedure of determining the effect of real estate attributes on land function with the use of the rough set theory

The experimental material comprised a set of data from 121 land property transactions conducted on the Olsztyn market between January 2007 and October 2008. To describe the procedure of investigating the effect that real estate attributes have on land function, each site was described with a minimal set of attributes which are usually taken into account by local government officers to determine the type of land use. Real estate prices, marked successively as $\mathbf{c_1, c_2, c_3, c_4, c_5, c_6}$ (table 1), are conditional attributes. Real estate function **d** is a decisional attribute.

Table 1

Conditional attributes and a decisional attribute applied in the study

Conditional attributes						Decisional attribute
c_1	**c_2**	**c_3**	**c_4**	**c_5**	**c_6**	**d**
Area	Location	Utilities	Attractiveness	Accessibility	Price	Function

Source: own study.

The above attributes were evaluated on the following scale:

c_1 - land area in m^2,

c_2 - location, encoded based on the following criteria: 1-inconvenient, 2-average, 3-convenient, 4-highly convenient,

c_3 - utilities supply, encoded based on the following criteria: 1-none, 2-partial supply, 3-full supply,

c_4 - attractiveness, encoded based on the following criteria: 1-low, 2-average, 3-high,

c_5 - transport accessibility: 1-poor, 2-average, 3-satisfactory, 4-highly satisfactory,

c_6 - price per m^2 of land as in October 2008,

d – function, encoded based on the following criteria: 1-single-family housing, 2-high-rise housing, 3-commercial, 4-transportation, 5-industrial, 6-recreational with development options, 7-other, 8-warehouses and storage facilities, 9-health care facilities.

At the next stage, the values of each real estate attribute were grouped according to the indiscernibility relation based on the rough set theory (PAWLAK 1982, 1991). For the purpose of analyzing this highly specific set of real estate data and different scales (ratio scale, ordinal scale, interval scale and nominal scales) for attribute assessment, the classical rough set theory was expanded to include the value tolerance relation formula. The formula, developed and discussed by STEFANOWSKI AND TSOUKIAS (2000) and STEFANOWSKI (2001), has been in real market analyses by D'AMATO (2006, 2007, 2008), RENIGIER – BIŁOZOR (2008a, 2008b) and RENIGIER-BIŁOZOR, BIŁOZOR (2007, 2008).

The classical rough set theory is based on the indiscernibility relation as a crisp equivalence relation, i.e. two real estate sites will be indiscernible only if characterized by similar attributes. When the rough set theory is expanded to include the value tolerance relation, the upper and lower approximation of the data set can be determined at different levels of the indiscernibility relation. The above relation can be expressed with the following formula:

$$R_j(x,y) = \frac{\max\left(0, \min\left(c_j(x), c_j(y)\right) + k - \max\left(c_j(x), c_j(y)\right)\right)}{k} \tag{1}$$

where: $R_j(x,y)$ - relation between sets with membership function [0,1]

$c_j(x), c_j(y)$ - variable of the analyzed real estate

k - coefficient of standard deviation in the attribute set of a given real estate

The formula is used to compare two data sets, i.e. two sites in this case, and the obtained result within the 0-1 range marks the level of the indiscernibility relation. If coefficient *k* from the above formula is the standard deviation (as cited in D'AMATO 2006) of the analyzed attribute, similarity (indiscernibility) matrices for coefficient *k* are determined separately for each real estate attribute. A sample matrix for the land area attribute is presented in table 2.

Table 2

Value tolerance relation matrix for the land area attribute

area	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**...**	**121**
1	1	0.97	0.68	0.96	0.85	0.96	0.96	0.90	0.84	0.91	0.97	0.76	0.91	0.73	0.88		0.95
2	0.97	1	0.71	0.93	0.88	0.99	0.99	0.87	0.87	0.94	1.00	0.79	0.93	0.76	0.90		0.98
3	0.68	0.71	1	0.64	0.83	0.72	0.72	0.58	0.84	0.77	0.71	0.92	0.77	0.95	0.80		0.73
4	0.96	0.93	0.64	1	0.81	0.92	0.92	0.94	0.80	0.87	0.93	0.72	0.87	0.69	0.84		0.91
5	0.85	0.88	0.83	0.81	1	0.90	0.89	0.75	0.99	0.94	0.88	0.91	0.95	0.88	0.98		0.90
6	0.96	0.99	0.72	0.92	0.90	1	1.00	0.86	0.89	0.95	0.99	0.80	0.95	0.77	0.92		0.99
7	0.96	0.99	0.72	0.92	0.89	1.00	1	0.86	0.89	0.95	0.99	0.80	0.95	0.77	0.92		0.99
8	0.90	0.87	0.58	0.94	0.75	0.86	0.86	1	0.74	0.81	0.87	0.66	0.81	0.63	0.78		0.85
9	0.84	0.87	0.84	0.80	0.99	0.89	0.89	0.74	1	0.93	0.87	0.92	0.94	0.88	0.97		0.89
10	0.91	0.94	0.77	0.87	0.94	0.95	0.95	0.81	0.93	1	0.94	0.85	1.00	0.82	0.97		0.96
11	0.97	1.00	0.71	0.93	0.88	0.99	0.99	0.87	0.87	0.94	1	0.79	0.93	0.76	0.90		0.98
12	0.76	0.79	0.92	0.72	0.91	0.80	0.80	0.66	0.92	0.85	0.79	1	0.85	0.97	0.88		0.81
13	0.91	0.93	0.77	0.87	0.95	0.95	0.95	0.81	0.94	1.00	0.93	0.85	1	0.82	0.97		0.96
14	0.73	0.76	0.95	0.69	0.88	0.77	0.77	0.63	0.88	0.82	0.76	0.97	0.82	1	0.85		0.78
15	0.88	0.90	0.80	0.84	0.98	0.92	0.92	0.78	0.97	0.97	0.90	0.88	0.97	0.85	1		0.93
...																	
121	0.95	0.98	0.73	0.91	0.90	0.99	0.99	0.85	0.89	0.96	0.98	0.81	0.96	0.78	0.93		1

Source: own study.

In the next step of the attribute validity analysis, the results generated by the above matrices are summed up and the sum matrix is determined based on the following assumption:

$$R_j(x,p) = \max\left(\sum_{j=1}^{n} R_j(x,p)\right) \quad (2)$$

where R_j is the value tolerance relation, x is the attribute of the analyzed real estate, p is the attribute relating to the conditional part of the investigated decision rule, n is the number of real estate attributes in the conditional part of the decision rule. A sample sum matrix is presented in table 3.

Table 3

Matrix of sums of real estate values determined based on the value tolerance relation matrix of each attribute

sum	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**...**	**121**
1	**6.00**	2.87	3.58	1.79	2.41	2.51	2.51	1.44	2.28	2.31	3.37	1.09	2.23	2.06	2.20		1.44
2	2.87	**6.00**	4.71	4.87	3.54	**5.64**	**5.64**	2.52	2.41	4.44	3.49	2.22	3.36	3.19	4.33		2.57
3	3.58	4.71	**6.00**	3.57	2.48	4.38	4.38	2.22	1.37	4.27	3.20	2.35	3.20	3.38	4.23		2.32
4	1.79	4.87	3.57	**6.00**	3.53	4.64	4.64	3.65	3.40	3.43	2.49	2.21	2.36	2.18	3.33		2.56
5	2.41	3.54	2.48	3.53	**6.00**	3.90	3.89	2.74	3.87	2.79	3.72	3.68	2.72	2.65	2.75		3.83
6	2.51	**5.64**	4.38	4.64	3.90	**6.00**	**6.00**	2.84	2.76	4.80	3.82	2.58	3.72	3.54	4.69		2.93
7	2.51	**5.64**	4.38	4.64	3.89	**6.00**	**6.00**	2.84	2.76	4.79	3.83	2.58	3.72	3.54	4.69		2.93
8	1.44	2.52	2.22	3.65	2.74	2.84	2.84	**6.00**	2.64	3.67	3.73	3.45	3.59	2.42	3.56		3.80
9	2.28	2.41	1.37	3.40	3.87	2.76	2.76	2.64	**6.00**	1.90	2.83	1.81	1.83	1.78	1.86		1.84
10	2.31	4.44	4.27	3.43	2.79	4.80	4.79	3.67	1.90	**6.00**	4.93	3.78	4.93	3.75	**5.90**		3.87
11	3.37	3.49	3.20	2.49	3.72	3.82	3.83	3.73	2.83	4.93	**6.00**	3.72	4.87	3.69	4.84		3.89
12	1.09	2.22	2.35	2.21	3.68	2.58	2.58	3.45	1.81	3.78	3.72	**6.00**	4.85	3.97	3.88		**5.65**
13	2.23	3.36	3.20	2.36	2.72	3.72	3.72	3.59	1.83	4.93	4.87	4.85	**6.00**	4.82	4.97		4.80
14	2.06	3.19	3.38	2.18	2.65	3.54	3.54	2.42	1.78	3.75	3.69	3.97	4.82	**6.00**	3.85		3.62
15	2.20	4.33	4.23	3.33	2.75	4.69	4.69	3.56	1.86	**5.90**	4.84	3.88	4.97	3.85	**6.00**		3.76
...																	
121	1.44	2.57	2.32	2.56	3.83	2.93	2.93	3.80	1.84	3.87	3.89	**5.65**	4.80	3.62	3.76		**6.00**

Source: own study.

In the next step of the discussed procedure, abstract classes were determined for each indiscernibility relation at a corresponding level of similarity between real estate sites. A similarity level of 85% was adopted in view of the specific features of the real estate market, the number of analyzed sites and the diversified method of encoding various attributes. The above implies that if 6 conditional attributes are applied in the analysis, indiscernible (similar) sites will be sites whose sum resulting from value tolerance relation matrices (formula 2) is 5.1 and higher (6 x 85% = 5.1). Those sums are indicated in bold type in table 3.

Abstract (indiscernibility) classes were then set for condition attributes, and in this case, they are equal to the number of sites input for analysis, i.e. 121. For example, the following abstract classes will be set for the first five sites:

I – 1

II – 2, 6, 7, 82
III – 3
IV – 4
V – 5, 81 .....

In the next step, the coverage and accuracy of approximation were determined for sets from the family of decisional attributes. For this purpose, the entire real estate set was divided into 9 decision sub-groups (corresponding to 9 land use functions, as per table 1) where coverage indicators where determined. Sample calculations for decisional attribute no. 1, i.e. single-family housing function, are presented in table 4.

Table 4

Approximation of classification of sets from the family of decisional attributes for function 1 – single-family housing

<table>
<tr><td>Decisional attribute no. 1</td><td>No. of sites in indisc. class of decisional attributes</td><td>NO. OF LOWER APPROXIM. SITES (classical theory)</td><td>No. of sites from upper approxim. (classical theory)</td><td>No. of positive cover sites</td><td>No. of conditional attributes from the set's boundary area</td><td>No. of conditional attributes from lower approxim.</td><td>No. of sites from lower approx. (non-repeatable)</td><td>No. of sites from upper approxim. (non-repeatable)</td></tr>
<tr><td>column</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>approximation results</td><td>38</td><td>34</td><td>132</td><td>101</td><td>18</td><td>20</td><td>26</td><td>51</td></tr>
<tr><td>C-accuracy of approximation (classical theory) column 2 divided by column 3</td><td></td><td colspan="2">34/132 = 0.26</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>C-cover of approximation (classical theory) col. 6 / col. 1</td><td colspan="6">20/38 = 0.53</td><td></td><td></td></tr>
<tr><td>C-accuracy of approximation (authors) col. 7/ col. 8</td><td></td><td></td><td></td><td></td><td></td><td></td><td colspan="2">26/51 = 0,51</td></tr>
<tr><td>C-cover of approximation (authors) col. 7 / col. 1</td><td colspan="7">26/38 = 0.68</td><td></td></tr>
<tr><td>C-positive approximation (authors) col. 4 / col. 3</td><td></td><td></td><td colspan="2">101/132 = 0.76</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="9">sum = 0.26 + 0.53 + 0.51 + 0.68 + 0.76 = 2.74</td></tr>
</table>

Source*: own study.*

Since the indicators of coverage and accuracy of approximation of the rough set theory fit classical theory assumptions, the authors of this study proposed additional coverage and accuracy indicators which are better adapted to the value tolerance relation (italicized in the table). The coverage of the entire abstract set for a given decisional attribute was estimated by summing up the value of each indicator, as per table 4. A similar procedure was applied to the 8 remaining decisional attribute types.

To estimate each attribute's validity for other land functions, the entire procedure was repeated from the moment of calculating matrix sums, by excluding every successive attribute and observing the changes induced by coverage indicators in the set of analyzed sites. A sample sum matrix without the land area attribute is presented in table 5. Since the number of investigated attributes is equal to 5 (without successive attributes), indiscernible (similar) sites will be sites whose sum resulting from value tolerance relation matrices (formula 2) is 4.25 and higher (5 x 85% = 4.25). Those sums are indicated in bold type in table 5.

Table 5

Matrix of sums of real estate values without the land area attribute

without area	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**...**	**121**
1	**5.00**	1.90	2.90	0.84	1.56	1.56	1.56	0.54	1.43	1.40	2.39	0.33	1.33	1.33	1.33		0.49
2	1.90	**5.00**	4.00	3.94	2.66	**4.66**	**4.66**	1.64	1.53	3.50	2.50	1.43	2.43	2.43	3.43		1.59
3	2.90	4.00	**5.00**	2.94	1.66	3.66	3.66	1.64	0.53	3.50	2.50	1.43	2.43	2.43	3.43		1.59
4	0.84	3.94	2.94	**5.00**	2.72	3.72	3.72	2.71	2.60	2.56	1.56	1.50	1.49	1.49	2.49		1.65
5	1.56	2.66	1.66	2.72	**5.00**	3.00	3.00	1.99	2.88	1.84	2.84	2.78	1.77	1.77	1.77		2.93
6	1.56	**4.66**	3.66	3.72	3.00	**5.00**	**5.00**	1.99	1.88	3.84	2.84	1.78	2.77	2.77	3.77		1.93
7	1.56	**4.66**	3.66	3.72	3.00	**5.00**	**5.00**	1.99	1.88	3.84	2.84	1.78	2.77	2.77	3.77		1.93
8	0.54	1.64	1.64	2.71	1.99	1.99	1.99	**5.00**	1.89	2.86	2.85	2.79	2.79	1.79	2.79		2.95
9	1.43	1.53	0.53	2.60	2.88	1.88	1.88	1.89	**5.00**	0.97	1.96	0.90	0.89	0.89	0.89		0.95
10	1.40	3.50	3.50	2.56	1.84	3.84	3.84	2.86	0.97	**5.00**	4.00	2.93	3.93	2.93	**4.93**		2.91
11	2.39	2.50	2.50	1.56	2.84	2.84	2.84	2.85	1.96	4.00	**5.00**	2.94	3.93	2.93	3.93		2.91
12	0.33	1.43	1.43	1.50	2.78	1.78	1.78	2.79	0.90	2.93	2.94	**5.00**	4.00	3.00	3.00		**4.84**
13	1.33	2.43	2.43	1.49	1.77	2.77	2.77	2.79	0.89	3.93	3.93	4.00	**5.00**	4.00	4.00		3.84
14	1.33	2.43	2.43	1.49	1.77	2.77	2.77	1.79	0.89	2.93	2.93	3.00	4.00	**5.00**	3.00		2.84
15	1.33	3.43	3.43	2.49	1.77	3.77	3.77	2.79	0.89	**4.93**	3.93	3.00	4.00	3.00	**5.00**		2.84
....																	
121	0.49	1.59	1.59	1.65	2.93	1.93	1.93	2.95	0.95	2.91	2.91	**4.84**	3.84	2.84	2.84		**5.00**

Source*: own study.*

Subsequent stages of the procedure are identical to those which have been performed above for all sites. The procedure ends with an estimation of coverage results after every successive attribute has been removed – table 6a, b, c, d, e, f, g, h,

i. In line with the rough set theory, attributes in respect of which the sum of indicators was near, equal or higher than the value of the indicator accounting for all attributes were considered as insignificant (italicized in tables). The adopted indicators are consistent with table 4.

Table 6a

Approximation of classification of sets for function 1 – single-family housing

Decisional attribute no. **1**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	*0.26*	0.19	0.09	**0.18**	**0.23**	0.10	0.17
C-coverage of approx. - classical	*0.53*	0.47	0.37	**0.39**	**0.47**	0.39	0.47
C-accuracy of approx. - authors	*0.51*	0.33	0.26	**0.37**	**0.46**	0.25	0.31
C-coverage of approx. - authors	*0.68*	0.50	0.5	**0.68**	**0.66**	0.5	0.47
C-positive approx. - authors	*0.76*	0.71	0.74	**0.80**	**0.75**	0.58	0.68
Sum	*2.74*	2.2	1.96	**2.42**	**2.57**	1.82	2.1

* After removing the land area attribute

Source: own study.

Table 6b

Approximation of classification of sets for function 2 – high-rise housing

Decisional attribute no. **2**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	*1*	0.86	0.54	0.62	0.76	0.83	0.83
C-coverage of approx. - classical	*1*	0.92	0.69	0.85	0.85	0.92	0.92
C-accuracy of approx. - authors	*1*	0.75	0.53	0.48	0.73	0.75	0.75
C-coverage of approx. - authors	*1*	0.92	0.77	0.85	0.85	0.92	0.92
C-positive approx. - authors	*1*	0.89	0.75	0.68	0.84	0.87	0.87
Sum	*5*	4.34	3.28	3.48	4.03	4.29	4.29

Source: own study.

Table 6c

Approximation of classification of sets for function 3 – commercial

Decisional attribute no. **3**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	***0.47***	0.26	0.09	0.12	**0.56**	0.05	0.15
C-coverage of approx. - classical	***0.75***	0.5	0.37	0.37	**0.87**	0.37	0.5
C-accuracy of approx. - authors	***0.4***	0.35	0.09	0.16	**0.5**	0.12	0.18
C-coverage of approx. - authors	***0.75***	0.5	0.37	0.37	**0.87**	0.37	0.5
C-positive approx. - authors	***0.59***	0.43	0.34	0.37	**0.62**	0.24	0.38
Sum	***2.96***	2.04	1.26	1.39	**3.42**	1.15	1.71

Source: own study.

Table 6d

Approximation of classification of sets for function 4 – transportation

Decisional attribute no. **4**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	***0.9***	0.82	0.2	0.08	**0.86**	0.02	**0.83**
C-coverage of approx. - classical	***0.81***	0.61	0.38	0.27	**0.69**	0.19	**0.73**
C-accuracy of approx. - authors	***0.57***	0.4	0.49	0.38	**0.46**	0.09	**0.43**
C-coverage of approx. - authors	***0.81***	0.64	0.81	0.58	**0.69**	0.19	**0.73**
C-positive approx. - authors	***0.93***	0.91	0.83	0.89	**0.91**	0.7	**0.9**
Sum	***4.02***	3.38	2.71	2.2	**3.61**	1.19	**3.62**

Source: own study.

Table 6e

Approximation of classification of sets for function 5 - industrial

Decisional attribute no. **5**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	***1***	0.62	0.11	0.11	**0.5**	0.12	0.05
C-coverage of approx. - classical	***1***	0.75	0.25	0.25	**0.75**	0.25	0.25
C-accuracy of approx. - authors	***1***	0.5	0.17	0.17	**0.43**	0.25	0.07
C-coverage of approx. - authors	***1***	0.75	0.25	0.25	**0.75**	0.75	0.25
C-positive approx. - authors	***1***	0.75	0.67	0.67	**0.6**	0.42	0.33
Sum	***5***	3.37	1.45	1.45	**3.03**	1.79	0.95

Source: own study.

Table 6f

Approximation of classification of sets for function 6 - recreational

Decisional attribute no. **6**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	***0.31***	**0.4**	0.19	0.31	**0.46**	**0.5**	**0.27**
C-coverage of approx. - classical	***0.56***	**0.56**	0.56	0.56	**0.67**	**0.67**	**0.56**
C-accuracy of approx. - authors	***0.5***	**0.5**	0.29	0.71	**0.9**	**0.7**	**0.45**
C-coverage of approx. - authors	***0.56***	**0.56**	0.56	0.56	**1**	**0.78**	**0.56**
C-positive approx. - authors	***0.86***	**0.85**	0.52	0.86	**0.89**	**0.9**	**0.76**
Sum	***2.79***	**2.87**	2.12	3	**3.92**	**3.55**	**2.6**

Source: own study.

Table 6g

Approximation of classification of sets for function 7 - other

Decisional attribute no. **7**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	***0.76***	**0.73**	0.1	0.55	**0.61**	0.24	0.17
C-coverage of approx. - classical	***0.83***	**0.83**	0.33	0.78	**0.78**	0.56	0.5
C-accuracy of approx. - authors	***0.71***	**0.6**	0.15	0.47	**0.64**	0.18	0.33
C-coverage of approx. - authors	***0.83***	**0.83**	0.33	0.83	**0.78**	0.56	0.56
C-positive approx. - authors	***0.86***	**0.83**	0.43	0.72	**0.77**	0.45	0.61
Sum	***3.99***	**3.82**	1.34	3.35	**3.58**	1.99	2.17

Source: own study.

Table 6h

Approximation of classification of sets for function 8 – warehouses and storage facilities

Decisional attribute no. **8**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	*1*	0.33	0.14	**1**	**1**	**1**	0.57
C-coverage of approx. - classical	*1*	0.67	0.33	**1**	**1**	**1**	0.67
C-accuracy of approx. - authors	*1*	0.2	0.25	**1**	**1**	**1**	0.5
C-coverage of approx. - authors	*1*	0.67	0.33	**1**	**1**	**1**	0.67
C-positive approx. - authors	*1*	0.42	0.71	**1**	**1**	**1**	0.71
Sum	**5**	2.29	1.76	**5**	**5**	**5**	3.12

Source: own study.

Table 6i

Approximation of classification of sets for function 9 – health care facilities

Decisional attribute no. **9**	*all attributes*	W/t area*	W/t loc.	**W/t utilit.**	**W/t attract.**	W/t access.	W/t price
C-accuracy of approx. – classical	0	0	0	0	0	0	0
C-coverage of approx. - classical	0	0	0	0	0	0	0
C-accuracy of approx. - authors	0	0	0	0	0	0	0
C-coverage of approx. - authors	0	0	0	0	0	0	0
C-positive approx. - authors	0.67	0.67	0.4	0.15	0.67	0.5	0.67
Sum	-	-	-	-	-	-	-

Source: own study.

The results for decisional attribute no. 9 were rejected – if any of the indicators equal zero, a given data set is completely discernible, i.e. it corresponds to other decisional attributes. Decision no. 9 does not show any individual dependencies in relation to conditional attributes. The above can be attributed to the fact that this decision group features only two real estate sites.

The above results indicate that the choice of function is affected by attributes which, in line with the rough set theory, form a core of the attributes set, as illustrated by table 7 (marked with a cross).

Table 7

Core of decisional attributes

	Area	Location	Utilities	Attractivene ss	Accessibil ity	Price
Decisional attribute no. 1	X	X			X	X
Decisional attribute no. 2	X	X	X	X	X	X
Decisional attribute no. 3	X	X	X		X	X
Decisional attribute no. 4	X	X	X		X	
Decisional attribute no. 5	X	X	X	X	X	X
Decisional attribute no. 6		X	X			
Decisional attribute no. 7		X	X		X	X
Decisional attribute no. 8	X	X				X

Decisional attribute no. 9	-	-	-	-	-	-

Source*: own study.*

In line with the rough set theory, the following attributes affected the choice of function for a given real estate set:

- single-family housing function – area, location, transport accessibility and price,
- high-rise housing function – area, location, utilities supply, attractiveness, transport accessibility and price,
- commercial function – area, location, utilities supply, transport accessibility and price,
- transportation function – area, location, utilities supply, transport accessibility,
- industrial function – area, location, utilities supply, attractiveness, transport accessibility and price,
- recreational function – location, utilities supply,
- other function – location, utilities supply, transport accessibility and price,
- warehouses and storage facilities function – area, location and price.

3. Conclusions

The practical problem of data exploration in spatial development results mainly from the non-homogeneity of real estate (no two properties are identical), the presence of various attributes describing real estate (both qualitative and quantitative), a highly diversified access to information on various segments of the real estate market and, frequently, the lack of awareness concerning the dependencies that exist between real estate attributes and market actors. Combined with information overload, imprecision, measurement errors and the unavailability of certain types of information, the above factors directly impact the effectiveness of the chosen investigation method and the quality of decisions.

The discussed procedure for assessing the significance of real estate attributes in the process of selecting land functions poses an alternative to statistical solutions, such as the multiple linear regression model which fails to produce satisfactory results for the analyzed data set, as illustrated by Table 8.

The above shows that the likehood of the regression model is around 20%, and attributes such as land area and price do not affect real estate value at a significance level of 0.05. It can be concluded that the data set is too diverse and it does not demonstrate a linear relationship between cause and effect. At such low likehood of the regression model, the significance of individual real estate attributes is difficult to investigate.

The method proposed in this study supports the evaluation of the significance of each attribute's effect on different land functions without the need to build special models. Based simply on Boolean logic, i.e. if condition, then decision, attributes which significantly affect the decision on real estate's function are determined for a given set of data. With the use of methods based on the rough set theory, the relevant observations "speak for themselves", and they are not adjusted in any respect, neither before the proposed method is applied, nor during analyses. Unlike regression models, the discussed model is not limited by the size of the set of representative observations (which may contain a very low or a very high number of observation samples) and it does not enforce a complex set of rules for controlling the investigated attributes and analysis results.

Table 8

Results of a multiple linear regression model for a set of 121 land transactions

Summary of a dependent variable regression: function (sets) R= 0.45584280 R2= 0.20779266 Adjust. R^2= 0.16609754 F(6.114)=4.9836 p						
	BETA	St. error	B	St. error	t(114)	p level
Absolute term			3.70323	1.121583	3.30179	0.001283
Location	-0.337753	0.094279	-0.99824	0.278644	-3.58248	0.000502
Utilities	-0.264152	0.086728	-1.09065	0.358089	-3.04575	0.002884
Area	-0.045136	0.086181	-0.00004	0.000081	-0.52374	0.601476
Attractivn.	0.197475	0.094109	0.98346	0.468677	2.09837	0.038081
Accessibility	0.281441	0.092350	0.66104	0.216909	3.04755	0.002868
Price	0.000693	0.098913	0.00001	0.001080	0.00700	0.994423

Source*: own study.*

To conclude, the applied theory and the proposed procedure are recommended mostly for "rare", imprecise data and for less effective real estate markets.

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